Dec. 19, 05

Physics 105. Classical physics

Solution set for Final Exam (2005)

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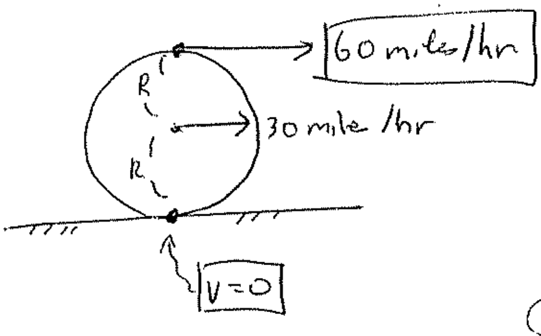
pg 1

Questions

Q1



To see this, let the point of contact on the ground be the pivot. Then by $V(r) = r\omega$,

① At origin $(r=0)$: $V(0)=0$

② At center $(r=R)$: $V(R) = R\omega = 30$ miles/hr

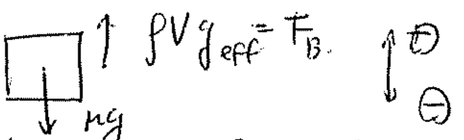
③ At rim $(r=2R)$: $V(2R) = 2R\omega = 60$ mile/hr.

Q2 When an elevator accelerates (up/down) with acceleration "a", then effective "g"$_{eff}$ is: $g_{eff} = g + a$ ($a>0$ accelerate up, $a<0$ accelerate down.)

Thus $p(h) = \rho h (g+a)$ ← pressure at depth h.

Hence when elevator accelerate upwards $(a>0)$, $F_{Buoyant} = \rho V (g+a)$ $V \equiv$ volume of submerged portion of wood.

∴ by Archimede's principle:

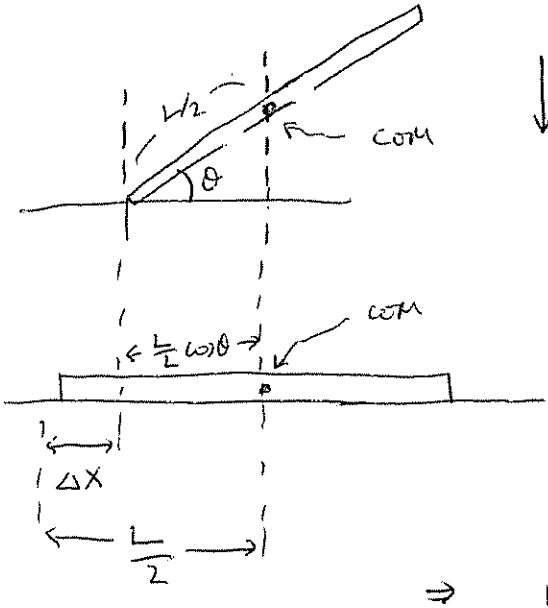


Notice that the wood can remain afloat since
$$\rho V g_{eff} - Mg = \rho V(g+a) - Mg = \underbrace{(\rho V g - Mg)}_{=0} + \underbrace{\rho V a}_{=Ma}$$

∴ $F_{net} = Ma$ with the wood afloat as it should.

$\boxed{\text{So the wood remains afloat}}$

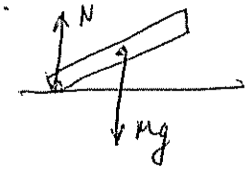
Q3



<u>Key idea</u>: Since ground is frictionless, ground only exerts upward normal force.

$\Rightarrow$ There's <u>no</u> horizontal force on stick

$\Rightarrow$ x-coordinate of position ~~remains~~ of com remains unchanged.



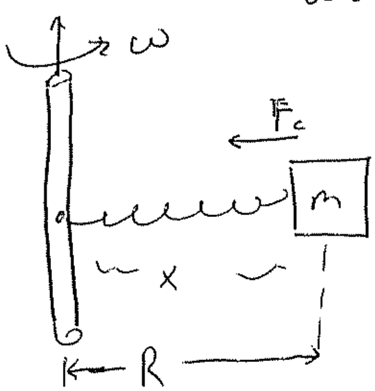
$$\Rightarrow \quad \frac{L}{2} = \frac{L}{2}\cos\theta + \Delta x$$

$$\Rightarrow \quad \boxed{\Delta x = \frac{L}{2}(1-\cos\theta)}$$

(The lower end of rod slips <u>back</u> (i.e. to left) by distance Δx.)

Q4

In absence of gravity, (say in deep space / space orbiting Earth), take a stick, and spin it about its axis at angular frequency ω. A spring is attached to the rod, and an ~~mass~~ unknown mass m is attached at the other end. The spring stretches ~~by~~ from equilibrium by an amount x, where:



<u>outer space</u>

$$F_c = \frac{mv^2}{R} = kx$$

(strictly speaking, $R \neq x$ since the rest length of spring is non-zero)

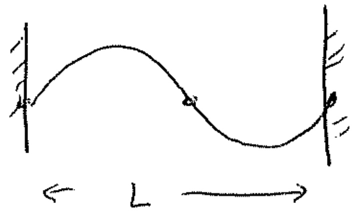
$$\Rightarrow \quad \boxed{m = \frac{kRx}{v^2} = \frac{kx}{R\omega^2}}$$

Thus, can figure out m if you have a spring with a known k, and measure R, & x and ω.

Q5

$\frac{\lambda_1}{2} = L$

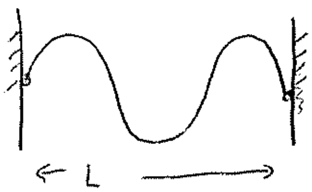
(lowest resonant f.)



$\lambda_2 = L$

(2nd lowest resonant f.)

and,



$\frac{3\lambda_3}{2} = L$

(3rd lowest f.)

, and $f\lambda = V$

material dependent $V = \sqrt{\frac{T}{\rho}}$ which is kept fixed.

The pattern we see is $\frac{n\lambda_n}{2} = L \Rightarrow f_n = \frac{nV}{2L}$

$\therefore\ f_3 = \frac{3V}{2L} = \frac{3}{2} f_2 = \frac{3}{2}(880\,\text{Hz}) = \boxed{1320\ \text{Hz}}$ ← third lowest resonance

Q6.

$[R] = L \qquad [\rho] = ML^{-3} \qquad [\sigma] = \left[\frac{F}{L}\right] = MT^{-2}$

surface tension (force/length)

Hence:

$$[f] = T^{-1} = [\rho^{\alpha}\sigma^{\beta}R^{\gamma}] = \underbrace{(M^{\alpha+\beta})}_{\alpha = -\beta}\underbrace{(L^{-3\alpha+\gamma})}_{\gamma = 3\alpha}(T^{-2\beta})$$

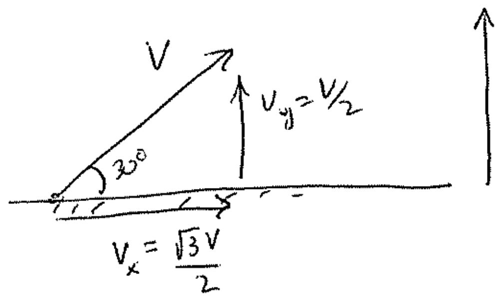
up to a multiplicative constant

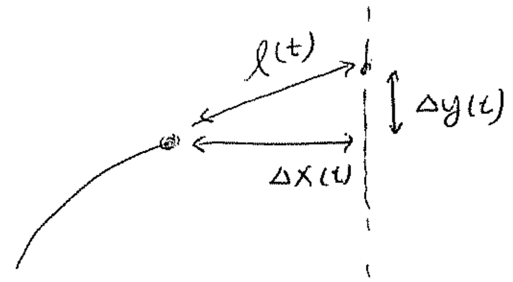
$\Rightarrow \beta = 1/2 \qquad \alpha = -1/2 \qquad \gamma = -3/2$

$\Rightarrow \boxed{f \propto \sqrt{\frac{\sigma}{\rho R^3}}}$

Problems

P1





$\Delta x(t) = L - \frac{\sqrt{3}}{2} v t \qquad \Delta y(t) = \left(vt - \frac{gt^2}{2}\right) - \left(\frac{v}{2}t - \frac{gt^2}{2}\right) = \frac{v}{2} t$

Thus; $\ell(t) = \sqrt{[\Delta x(t)]^2 + [\Delta y(t)]^2} = \sqrt{L^2 - \sqrt{3} v t L + \frac{3}{4} v^2 t^2 + \frac{v^2}{4} t^2}$

$= \sqrt{L^2 + v^2 t^2 - \sqrt{3} v t L}$ ← distance as a fn. of t.

Hence $0 = \frac{d\ell}{dt} = \frac{1}{2} \frac{(2v^2 t - \sqrt{3} v L)}{\sqrt{L^2 + v^2 t^2 - \sqrt{3} v t L}} \Leftrightarrow t_0 = \frac{\sqrt{3} L}{2V}$

← time at which $\ell(t) = \ell_{min}$

Hence: $\ell_{min} = \ell(t_0) = \sqrt{L^2 + \frac{3}{4} L^2 - \frac{3L^2}{2}}$

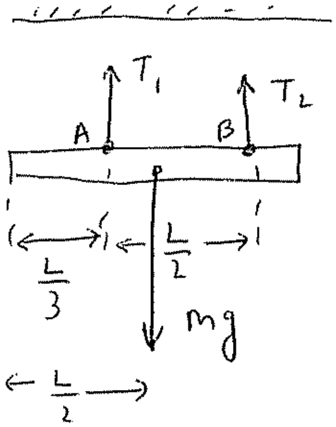
$= \boxed{\frac{L}{2}}$ ← minimal distance between projectiles

Subtlety: Above sol'n assumes implicitly that the speed v is large enough such that ~~the~~ neither of the 2 projectiles hit the ground before $t_0 = \frac{\sqrt{3} L}{2V}$. ⇒ For sufficiently small speeds, above sol'n is not valid. In fact, since the total time in air for the projectile launched at 30° (it comes down to ground sooner than the other) is $t_{tot} = \frac{V}{g}$, we need $\frac{V}{g} \geq t_0 = \frac{\sqrt{3} L}{2V}$

$\Leftrightarrow \boxed{V \geq \left(\frac{\sqrt{3} g L}{2}\right)^{1/2}}$ for above sol'n to be valid.

Note! You didn't have to calculate this on the exam. This is just extra info for you. ☐

P2.



Since in equilibrium: 2 conditions

$F_{net} = 0 \Rightarrow \quad T_1 + T_2 = mg \quad \ldots (1)$

$\tau_{net} = 0 \Rightarrow$ (About the origin = A):

about any origin.

$$0 = mg\left(\frac{L}{2} - \frac{L}{3}\right) - T_2 \frac{L}{2}$$

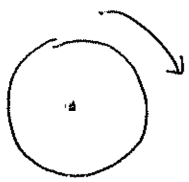
$$= \frac{mgL}{6} - \frac{T_2 L}{2}$$

$$\Rightarrow \boxed{T_2 = \frac{mg}{3}}$$

and from (1) we get

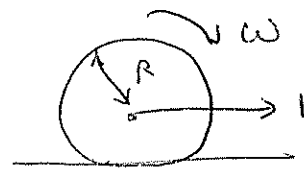
$$\boxed{T_1 = \frac{2mg}{3}}$$

P3



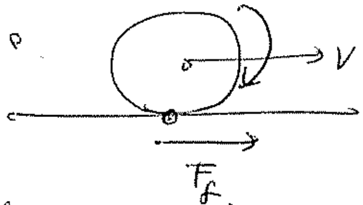
Initially

Afterwards



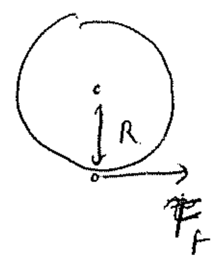
No slippage (i.e. there's friction on ground.)

The reason that center of mass speed V increases while angular speed decreases is that for a rolling motion on level ground, friction points in the same direction as the translational motion. i.e.



⇒ Due to work done by friction, you can't simply use energy conservation. Instead, use impulse.

During time Δt:



(About com):

$\tau = F_f R.$

$\Rightarrow \Delta L = (F_f \Delta t) R \leftarrow$ change in ang. momentum

$\Rightarrow I(\omega_0 - \omega) = (F_f \Delta t) R. \quad \ldots (1)$

And the change in linear momentum: $mV = (F_f \Delta t) \leftarrow$ "linear" impulse

Hence (1) becomes: $I(\omega_0 - \omega) = mVR$

over

No slippage condition: $R\omega = V$

thus: $I\left(\omega_0 - \frac{V}{R}\right) = mVR$

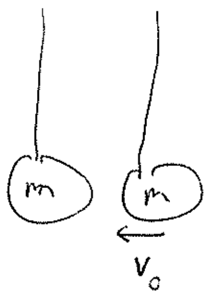
$\Rightarrow \cancel{m}\frac{\cancel{R^2}}{2}\left(\omega_0 - \frac{V}{R}\right) = \cancel{m}V\cancel{R}$ ← ∵ $I = \frac{mR^2}{2}$ for cylinder about its COM.

$\Rightarrow \frac{R\omega_0}{2} = \frac{3V}{2} \Rightarrow \boxed{\frac{R\omega_0}{3} = V}$

* linear speed of cylinder once slippage ends.

P4 At the instant the collision occurs, you can either use lin. momentum or ang. momentum conservation to get the speeds just after collision.

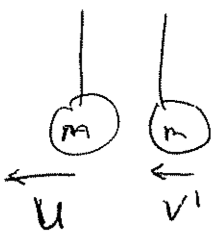
Just before:



$\frac{1}{2}mV_0^2 = mgL \Rightarrow V_0 = \sqrt{2gL}$

$P_{tot} = mV_0 = m\sqrt{2gL}$

Just after:



By momentum conservation, we have:

$\cancel{m}V_0 = \cancel{m}u + \cancel{m}V' \quad \ldots (1)$

Now, since the incident ball goes up to height $L/9$, we have:

$\frac{mV'^2}{2} = mg\frac{L}{9} \Rightarrow V' = \frac{\sqrt{2gL}}{3}$.

(1) becomes:

$\Rightarrow u = \frac{2\sqrt{2gL}}{3} \Rightarrow \frac{1}{2}mu^2 = mgh$

$\Rightarrow \frac{1}{\cancel{2}}\frac{4}{9}(\cancel{2}\cancel{g}L) = \cancel{g}h \Rightarrow \boxed{h = \frac{4L}{9}}$

and, $E_{tot,\,initial} = mgL = KE_{tot}$ just before collision.

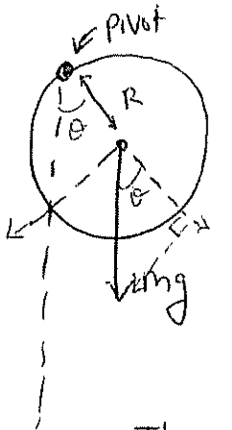
$$E_{tot,\,after} = mg\left(\frac{4L}{9}\right) + mg\left(\frac{L}{9}\right) = mg\,\frac{5L}{9}.$$

Hence $\boxed{\Delta E = \frac{4mgL}{9}}$ ← energy lost to heat.

$\therefore$ $\boxed{\frac{\Delta E}{E_{tot,\,initial}} = 4/9}$ ← fraction of KE converted to heat during collision.

$\boxed{P5}$ Ex of a physical pendulum.

We solve this by writing down the eq'n of motion. (i.e. $\tau_{net} = I\alpha$)



using the pivot as origin:

$$\tau = -(mg\sin\theta)R \approx -mgR\theta$$ ← ∵ $\sin\theta \approx \theta$ when θ small (via Taylor approx.)

$$I_{about\ pivot} = I_{com} + mR^2$$ ← parallel axis thm.

$$= \frac{mR^2}{2} + mR^2$$
$$= \frac{3mR^2}{2}$$

Thus: eq'n of motion is:

$$-\cancel{m}g\cancel{R}\theta = \left(\frac{3\cancel{m}R^{\cancel{2}}}{2}\right)\ddot{\theta}$$

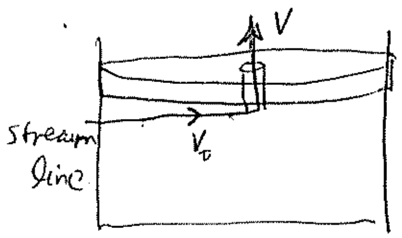
$$\Rightarrow \ddot{\theta} + \left(\frac{2g}{3R}\right)\theta = 0 \quad \text{(EOM)}$$

$\Rightarrow \omega^2 = \frac{2g}{3R}$ and since $T = \frac{1}{f} = \frac{2\pi}{\omega}$

we get: $\boxed{T = 2\pi\sqrt{\frac{3R}{2g}}}$ ← period of small oscillations.

P6. (PS8)

Use Bernoulli's law to the following streamline.



atmospheric pressure (at pressure P_0) (since exposed to atmosphere.)

V_0 (at pressure P)

Then: $$P + \frac{1}{2}\rho V_0^2 = P_0 + \frac{1}{2}\rho V^2$$

(No "$\rho g h$" terms since there two points are just adjacent to each other (so variation in height ≈ 0).

Now, $P = \frac{mg}{A} + P_0$.

Thus: $$\left(\frac{mg}{A} + P_0\right) + \frac{1}{2}\rho V_0^2 = P_0 + \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad \frac{mg}{A} + \frac{1}{2}\rho V_0^2 = \frac{1}{2}\rho V^2$$

← since hole is tiny compared to A, can assume $V_0 \ll V$ ($V_0 \approx 0$) much like the case in Torricelli's law

$$\Rightarrow \quad \boxed{\sqrt{\frac{2mg}{\rho A}} \approx V}$$

↳ "effusion" speed through hole.

☒