Quantum Mechanics I Problem Set 8
Due Wednesday, November 9, 2005

If you don't have a Mathematica License, please contact Andrew Blechman to get that.

1. Townsend Chapter 6, Problem 2.

2. Townsend Chapter 6, Problem 3.

3. Townsend Chapter 6, Problem 6.

4. Townsend Chapter 6, Problem 7.

5. Suppose $g(x)$ is a function with a single root at $x = x_1$, that is, $g(x_1) = 0$. Show that

$$\delta(g(x)) = \frac{1}{|g'(x_1)|}\delta(x - x_1) .$$

(Hint: Taylor expand around the point $x = x_1$ and use a formula from class.) Now generalize your result to a function $g(x)$ with n roots $\{x_1, \ldots, x_n\}$.

6. Consider a free particle whose position-space wave function is given at $t = 0$ by

$$\Psi(x,0) = \langle x|\psi\rangle = \begin{cases} A\,(a - |x|)\ , & \text{if } -a < x < a \\ 0\,, & \text{otherwise}\,, \end{cases}$$

where A and a are positive real constants.

a. Determine A by normalizing $\Psi(x,0)$.

b. Determine $\Phi(p,0) = \langle p|\psi\rangle$ at $t = 0$. Sketch it. Comment on the behavior of $\langle p|\psi\rangle$ for very small and very large values of a. How does this relate to the uncertainty principle?

d. Find $\Phi(p,t)$.

e. Find $\Psi(x,t)$. You don't have to do the last integral.

f. Use Mathematica to make sketches of the solution. Set $a = \hbar = 1$. Use the numerical integration command **NIntegrate[...]** to compute $\Psi(x,t)$. For numerical reasons it is useful to replace the integral over $-\infty < p < \infty$ with a pair of integrals, for negative and positive p, over the range $p_{\min} < |p| < p_{\max}$, and take $p_{\min} = 0.001$ and $p_{\max} = 25$.

Plot the probability density $|\Psi(x,t)|^2$, over the range $-4 < x < 4$, for the following values of t: $t = 0, 0.25, 0.50, 0.75, 0.1, 0.2, 0.5, 1, 2$. With Mathematica, this is easy to automate. You will see the wave packet spread out!

g. What is wrong with your plot at $t = 0$? Where did the extra wiggles come from? The extra wiggles are apparent for small values of t.