Quantum Mechanics I, Problem Set 9

Due Wednesday, November 16, 2005

1. Griffith, Chapter 2, Problem 1
2. Griffith, Chapter 2, Problem 22
3. Griffith, Chapter 2, Problem 33. You will see much of this in class Wednesday 11/9 and possibly into Monday.
4. Griffith, Chapter 2, Problem 43
5. Griffith, Chapter 2, Problem 45

6. The probability of finding a particle *anywhere* in the region $-\infty < x < \infty$ is given by

$$P = \int_{-\infty}^{\infty} \mathrm{d}x\, \Psi^*(x,t)\, \Psi(x,t)\,.$$

a. Use the Schrödinger Equation in position space to show that $\mathrm{d}P/\mathrm{d}t = 0$, so the total probability is conserved. You will need the fact that the Hamiltonian operator, $\hat{H} = \hat{p}^2/2m + V(x)$ is Hermitian.

b. Now let's assign a constant *imaginary* part to the potential energy, so

$$V(x) = V_0(x) - \mathrm{i}\Gamma\,,$$

with $\Gamma > 0$. Show that in this case

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{2\Gamma}{\hbar}\,P\,.$$

c. The solution for $P(t)$ is of the form $P(t) = P(0)e^{-t/\tau}$. Find τ, called the "lifetime", in terms of Γ. Note that the probability for finding the particle *anywhere* decays with time. This is a simple way to model an *unstable* particle.

d. Explain why it is not reasonable to take $V(x) = V_0(x) + \mathrm{i}\Gamma$ instead.