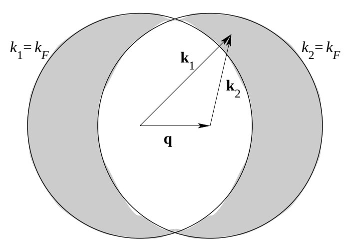
Condensed Matter Physics

Homework Assignment 11

Due date Friday, November 19



1*. Lindhard function in 3 dimensions.
(a) Compute the charge-density susceptibility of the Fermi gas at $T = 0$,

$$\chi(\mathbf{q}) = 2e^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{n_{\mathbf{k-q/2}} - n_{\mathbf{k+q/2}}}{E_{\mathbf{k-q/2}} - E_{\mathbf{k+q/2}}}.$$

Hint. The integration region spans two crescents shown above. The integration is most effectively done in elliptical coordinates $k_1 = |\mathbf{k} - \mathbf{q/2}|$, $k_2 = |\mathbf{k} + \mathbf{q/2}|$ and ϕ (the azimuthal angle). The volume element in these coordinates

$$\mathrm{d}^3 k = \frac{1}{q} \, k_1 \mathrm{d}k_1 \, k_2 \mathrm{d}k_2 \, \mathrm{d}\phi.$$

(The answer is given in Ch. 17 of *Ashcroft and Mermin.*)

(b) Although the susceptibility is finite everywhere, its first derivative diverges logarithmically at $q = 2k_F$. That surely means that the response in real space,

$$\chi(\mathbf{r}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\, \chi(\mathbf{q}) \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}},$$

will contain oscillations with the characteristic wavelength $\lambda = \pi/k_F$.

2. Specific heat of the Fermi liquid. You may find it helpful to consult Ch. 1 and 2 in *Statistical Physics*, v. 2, by Landau (a.k.a. volume 9 of *Landau and Lifshitz*). In what follows we neglect the spin-dependent interactions.

(a) The excitation energy of a quasiparticle in the Fermi liquid is

$$\varepsilon(\mathbf{p}) + \int \frac{\mathrm{d}^3p'}{(2\pi\hbar)^3}\, f(\mathbf{p}, \mathbf{p}')\, \delta n(\mathbf{p}'),$$

where $\delta n(\mathbf{p})$ is the deviation of the quasiparticle distribution from that in the ground state. Show that the thermal effects alone do not alter the excitation energy. In other words, the quasiparticle energy does not depend explicitly on temperature. [Assume that the density of states is constant, $g(\varepsilon) = g(\varepsilon_F)$.]

(b) Show that specific heat of the Fermi liquid differs from that of the Fermi gas with the same density and mass by the factor m^*/m.