Condensed Matter Physics

Homework Assignment 8

Due date Friday, October 29

1. Problem 2.1 in *Ashcroft and Mermin.*

2. Pauli paramagnetism. Obtain the spin contribution to magnetic susceptibility

$$\chi = \frac{1}{V}\frac{M}{B}$$

of the ideal Fermi gas for a weak magnetic field, $\mu_B B \ll k_B T$.

3*. The asterisk signifies that this problem is *for graduate students.* Those of you who are registered for 171.405 are welcome to try it for extra credit. Appendix C in *Ashcroft and Mermin* might come in handy.

(a) Show that heat capacity (at constant chemical potential and volume) can be obtained by differentiating the logarithm of the partition function:

$$C \equiv T\left(\frac{\partial S}{\partial T}\right)_{\mu,V} = \beta^2\left(\frac{\partial^2 \log Z}{\partial \beta^2}\right)_{\mu,V},$$

where $1/\beta = k_B T$. It helps to know that $-T\log Z = \Omega$, where

$$\Omega(T,\mu,V) = E - TS - \mu N, \qquad \mathrm{d}\Omega = -S\mathrm{d}T - N\mathrm{d}\mu - P\mathrm{d}V.$$

(b) For noninteracting fermions, any thermodynamic potential is simply a sum of independent potentials of all $\mathbf{k}$:

$$\log Z = \sum_{\mathbf{k}} \log\left(1 + e^{-\beta(\epsilon_{\mathbf{k}}-\mu)}\right) = V\int \log\left(1 + e^{-\beta(\epsilon-\mu)}\right) g(\epsilon)\mathrm{d}\epsilon.$$

By using the Sommerfeld expansion, deduce the fermion specific heat up to terms of order T^3. Compare the fermionic T^3 term to the specific heat of phonons at low temperatures.

Assigned reading: Chapter 2 and Appendix C in *Ashcroft and Mermin.*