Multiscale Modeling
Second Assignment. Due March 11, 2005

1. The differential equation $\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 0$ has been discretized as

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} + u\frac{T_i^n - T_{i-1}^n}{\Delta x} = 0$$

where i is the spatial index and n the time step.

a) Find the stability condition of the difference equation.

b) You will find that for some values of u the difference scheme is always unstable.

c) Can you find a little modification of the scheme that makes it stable?

2. Find the stability condition of the following difference equation:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} + u\frac{T_{i+1}^n - {}_{i-1}^n}{2\Delta x} = D\frac{T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}}{\Delta x^2}.$$

3. Calculate the cavity flow for Reynolds number Re=6.25, 25, 100, 400, 1600 using the code sor0.f that will be emailed to you. The code is downloaded from www.cavityflow.com and a few modifications have been made for your easy use. Please
   a. Plot the contours of the stream function for each case and you will see a primary vertex near center and possible sub vertexes near bottom two corners and up-left corners.
   b. Plot the center position of the primary vertex and the maximum of stream function as functions of *Re*.
   c. Change the grid number "N" in the code from 128 to 64, 32, and 16 for Re=100 and 400 to see the variation of the solution. Plot the center position of vertex and the maximum of stream function as functions of *Re* for the primary vertex.