Advanced Statistical Mechanics 171.703

Homework Assignment 11

Due date Friday, April 30

Assigned reading. Ch. 110–112, 118 in Landau; 14.3–14.4 in Pathria.

1. Thermodynamic fluctuations in a fluid. Prob. 14.1 in Pathria.

2. Johnson-Nyquist noise in a resistor. Consider fluctuations of charge Q in a metallic capacitor C shunted by a resistor R.

(a) Show that the probability distribution of having charges $\pm Q$ on the opposite plates is

$$\rho(Q) = \sqrt{Ck_BT/2\pi}\,\exp\left(-Q^2/2Ck_BT\right).$$

(b) The dynamics of charge is governed by Kirkhoff's law

$$R\dot{Q} = -Q/C + u(t),$$

where $u(t)$ is a random voltage (produced by the resistor) whose statistical properties resemble white noise:

$$\langle u(t)u(0)\rangle = \gamma\delta(t).$$

Relate the coefficient γ to temperature and the parameters of the circuit.

(c) Find the Fourier spectrum of current fluctuations

$$I(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\,\langle \dot{Q}(0)\dot{Q}(t)\rangle\, e^{i\omega t}.$$

3. Brownian motion in 1, 2, and 3 dimensions. In a large ensemble of particles, Brownian motion is equivalent to diffusion:

$$\partial n/\partial t = D\,\nabla^2 n,$$

where $n(\mathbf{r}, t)$ is the number density. Although the diffusion equation looks the same in any number of spatial dimensions d, the physics of Brownian motion in $d = 1$ is *very* different from that in $d = 3$. A particle randomly moving along a straight line ($d = 1$) will return to the starting point many times in the future. Not so in 3 dimensions [3].

(a) At $t = 0$, N particles are released at the origin: $n(\mathbf{r}, 0) = N\delta(\mathbf{r})$. Determine the density distribution at $t > 0$ (in an arbitrary number of dimensions).

(b) T seconds later, take a small sample of the medium of volume $V \ll (DT)^{d/2}$ at the origin. How many particles do you expect to catch?

(c) Repeat the process every T seconds *ad infinitum*. How many particles will you catch overall?

References

[1] J.B. Johnson, Phys. Rev. **32**, 97 (1928).

[2] H. Nyquist, Phys. Rev. **32**, 110 (1928).

[3] L. David, *The Parking Garage*, Seinfeld, Episode 23 (1991).