Advanced Statistical Mechanics 171.703

Homework Assignment 7

Due date Friday, March 26

Reading. Ch. 56–60 in Landau I, 1–3 in Landau II, 8.4 in Pathria.

1. Compressibility of a fluid. The isothermal compressibility of a fluid is defined as $\kappa_T = -\left(\partial \log n / \partial P\right)_T$. Show that

$$\kappa_T^{-1} = N \left(\frac{\partial P}{\partial N}\right)_{T,V} = \frac{N^2}{V} \left(\frac{\partial \mu}{\partial N}\right)_{T,V}.$$

2. Incompressible electron fluid in quantum Hall layers. Condiser an ideal Fermi gas confined to a plane in magnetic field $\mathbf{B}$ perpendicular to the plane. For simplicity, take spinless fermions. Compute the compressibility at low temperatures $k_B T \ll \hbar\omega_c$, where $\omega_c = eB/mc$ is the cyclotron frequency. Incompressibility of the electron fluid at certain densities is a prominent feature of the quantum Hall effect. See, e.g., G. Finkelstein *et al.*, Phys. Rev. B **61**, 16323 (2000) and G. Finkelstein, invited talk at KITP, Santa Barbara (2001).

3. Specific heat of the Fermi liquid. In what follows we neglect the spin-dependent interactions.

(a) The excitation energy of a quasiparticle in the Fermi liquid is

$$\varepsilon(\mathbf{p}) + \int \frac{\mathrm{d}^3 p'}{(2\pi\hbar)^3} f(\mathbf{p}, \mathbf{p}')\, \delta n(\mathbf{p}'),$$

where $\delta n(\mathbf{p})$ is the deviation of the quasiparticle distribution from that in the ground state. Show that the thermal broadening of the Fermi distribution alone do not alter the excitation energy. In other words, verify that the quasiparticle energy does not depend explicitly on temperature. [Assume that the density of states is constant, $\rho(\varepsilon) \approx \rho(\varepsilon_F)$.]

(b) Show that specific heat of the Fermi liquid differs from that of the Fermi gas with the same density by the factor m^*/m.

4. Fermi temperatures in physical systems. Prob. 8.8 in Pathria.