Homework 1 (due Mon Feb.13, 2006)
Prof. Andrei Gritsan, January 2006

This homework assignment covers selected topics in Chapters 1 and 2.

Problem 1 (similar to Problem 1.3 in the textbook)
(a) Find the momentum of the muon ($\mu^+$) in the reaction

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

in the rest frame of the pion ($\pi^+$). Express your answer in terms of the masses of the three particles and assume that neutrino mass $m_\nu$ is non-zero. (Hint: use 4-vectors);

(b) Show that under the assumption $m_\nu = 0$ you get the formula derived in class;

(c) Assuming that $m_\nu \ll m_\mu < m_\pi$, derive an approximate formula keeping the leading terms in $m_\nu^2$ and show that the muon momentum would be reduced by the fraction

$$\frac{\Delta p}{p} = \frac{m_\nu^2}{m_\pi^2} \times m_\nu^2$$

compared with the case of massless neutrino;

(d) Using the mass values in MeV from the Particle Data Group table (or the textbook), find the value of the coefficient in front of $m_\nu^2$ in the above formula.

Problem 2

The cosmic rays are formed in the upper atmosphere. Charged mesons (such as $\pi^+$) are produced in interactions of the primary cosmics rays (such as protons) in the air. Muons ($\mu^+$) and neutrinos are products of the decay of charged mesons (e.g. $\pi^+ \rightarrow \mu^+ + \nu_\mu$).

(a) Calculate the mean flight distance of $\pi^+$ with the energy of $E = 1$ GeV. What is the value in meters for the typical energy $E = 1$ GeV.

(b) Use the formula derived in (a) to calculate the mean flight distance of $\mu^+$ with the same energy $E = 1$ GeV.

(c) How different is the answer in (b) if $E = 4$ GeV?

(d) How different would be the answer in (b) if you ignore the relativistic effect of the different lifetime of a particle in the laboratory and the center-of-mass frames?

(Hint: use the Particle Data Group summary when necessary, you can use the values of $c\tau$ and $mc^2$ quoted in meters and MeV in the tables to simplify the calculations).

Problem 3 (similar to Problem 1.7)

Which of the following reactions or decays are allowed by conservation laws and which are forbidden. If forbidden, give the reason.

(a) $\pi^- \rightarrow \mu^- + \nu_\mu$
(b) $\mu^+ \rightarrow \pi^+ + \bar{\nu}_\mu$
(c) $\pi^+ + \pi^- \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu$
(d) $n + \nu_e \rightarrow e^- + p$
(e) $\pi^+ \rightarrow e^+ + e^- + e^+ + \nu_e$

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**Problem 4** (similar to Problem 2.5)

Draw Feynman diagrams (in terms of transitions at the quark level if hadrons are involved) for the following strong decays:

(a) $\rho^0 \rightarrow \pi^+ + \pi^-$  
(b) $\omega \rightarrow \pi^+ + \pi^- + \pi^0$  
(c) $\phi \rightarrow K^+ + K^-$  
(d) $\Delta^{++} \rightarrow p + \pi^+$

**Problem 5** (similar to Problem 2.5)

Draw Feynman diagrams (in terms of transitions at the quark level if hadrons are involved) for the following weak decays:

(a) $\pi^+ \rightarrow \mu^+ + \nu_\mu$  
(b) $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$  
(c) $\Lambda \rightarrow p + e^- + \bar{\nu}_e$  
(d) $\Omega^- \rightarrow \Lambda + K^-$

**Problem 6** (Problem 2.6)

Show that, in the process of pair conversion $\gamma \rightarrow e^+e^-$, it is impossible to conserve energy and momentum without the participation of another particle (a nucleus), as in Fig.2.1(e) in the textbook. Calculate the minimum transfer from this extra particle for a $\gamma$-ray of energy $E=h\nu$. Show that for $E = 1$ GeV, the minimum transfer is approximately 520 eV.