Homework 5 (due Mon April 17, 2006)  
Prof. Andrei Gritsan, April 2006

The first three problems in this homework assignment cover selected topics in Chapters 5 and 6. The last three problems cover selected topics in Chapters 7.

Problem 1

Assume that parton densities in the proton could be parameterized with the following functions:

\[ V_u^P(x) = u(x) - \bar{u}(x) = a_1 (1 - x)\rho x^{-1/2} \]
\[ V_d^P(x) = d(x) - \bar{d}(x) = a_2 (1 - x)^{\rho+1} x^{-1/2} \]

where \( x \) is the Bjorken variable (the fraction of the momentum of a nucleon carried by a quark) and \( \rho \) is a parameters.

(a) ensure the quantum numbers of the proton (charge 1, baryon number 1) and find the values of \( a_1 \) and \( a_2 \).

(b) using your answer in part (a) find the ratio of the average momentum carried by up quarks over the average momentum carried by the down quark. Ignore the presence of sea quarks in this calculation.

(c) again, using (a) and ignoring presence of sea quarks, calculate the average momentum carried by the quarks.

In (a) express your answer in terms of \( \Gamma \) functions and the parameter \( \rho \). In both (b) and (c) express your answer using parameter \( \rho \) only, and give a quantitative answer assuming \( \rho = 3 \).

Hint: use beta function decomposition in terms of gamma functions:

\[ B(x,y) = \int_0^1 t^{x-1} (1 - t)^{y-1} dt = \Gamma(x)\Gamma(y)/\Gamma(x + y) \]
\[ \Gamma(x + 1) = x\Gamma(x) \]

Problem 2

Which of the neutral mesons with the following \( J^{PC} \) quantum numbers are “exotic” (not possible for \( q\bar{q} \) state), which are possible “gluonium” states made of two bound gluons, and which are possible \( q\bar{q} \) states. More than one assignment is possible in each case. Explain when possible.

(a) 0-- , (b) 0+, (c) 0-, (d) 0++
(e) 1-, (f) 1+, (g) 1++, (h) 1--

Problem 3

Consider a beam of left-handed polarized fermions (spin \( \hbar/2 \)) scattering through angle \( \theta \). If the interaction flips the spin orientation of the fermion (it becomes right-handed), find the angular distribution of the scattered fermions.

Hint: you can use Appendix D of the textbook.

(continue to page 2 on the other side)
Problem 4

In the weak decays $\pi^+ \rightarrow e^+ \nu_e$ and $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$, what is the polarization (helicity) of the positron in each case (positive or negative). Hint: remember about the vector-axial vector form of the weak interaction; neutrinos have essentially zero mass for the purpose of this problem.

Problem 5

Show why observation of the process $\bar{\nu}_\mu e^- \rightarrow e^- \bar{\nu}_\mu$ constitutes unique evidence for neutral currents, whereas observation of $\bar{\nu}_e e^- \rightarrow e^- \bar{\nu}_e$ does not.

Problem 6

List at least three decays of the $\tau$ lepton. Draw Feynman diagrams in each case. If quarks are involved, do not worry about their hadronization and leave the diagrams at quark level (there is a long list of possible ways quarks can create hadrons in strong interactions, you can find it in the particle data group listing).