What is Space-Time?

• Space-time: combination of space and time
  3 dimensions of space (though this is being questioned)
  time is the 4th dimension

• It is a mathematical model to write physics laws
Space-Time

- Space-time in classical physics (Galilean transformation)
  ⇒ time is absolute and independent of space

- In Special Relativity (and later General)
  ⇒ time and space are related ⇒ time different in frames
Problem of Classical Physics (1900’s)

- Relativity and E&M looked inconsistent

relativity – same physics laws in all systems

E&M – Maxwell’s equations involve speed of light = \( c \)

Classically: speed of light different in different systems

classically \( c' = c - v \)

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Postulate of Special Relativity

• Einstein in 1905:
  – relativity valid – same physics laws in all systems
  – speed of light – same in all systems

• What is speed of light?
  \[ c = 299\,792\,458\ \text{m/s} \approx 3 \times 10^8\ \text{m/s} \]

• Breaks classical Newtonian mechanics
  \[ t = t' \]  
  \[ x = x' + v \times t' \]  
  \[ y = y' \]  
  \[ z = z' \]
Consequence of Einstein’s postulates

- Special Relativity

\[ t = \frac{t' + \frac{v}{c^2} \times x'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (5) \]

\[ x = \frac{x' + v \times t'}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (6) \]

\[ y = y', \quad (7) \]

\[ z = z'. \quad (8) \]
Did the trick work?

- A photon (light) will travel $x = c \times t$

$$x = (x' + v \times t')/\sqrt{1 - \frac{v^2}{c^2}} = c \times t$$

$$t = (t' + \frac{v}{c^2} \times x')/\sqrt{1 - \frac{v^2}{c^2}}$$

$\Rightarrow (x' + v \times t') = (ct' + \frac{v}{c} \times x')$

$\Rightarrow c' = \frac{x'}{t'} = \frac{c-v}{1-v/c} \equiv c \Rightarrow$ same speed of light
Consequences

• **Length contraction**

When a body moves with speed $v$ relative to the observer, its length is contracted in the direction of motion by

$$\times \sqrt{1 - \frac{v^2}{c^2}}$$

• **Time dilation**

When a clock moves with speed $v$ relative to the observer, its rate is measured to have slowed down by

$$\times \sqrt{1 - \frac{v^2}{c^2}}$$
Length contraction

- Consider a moving rod of length $L$ in our frame
  
  $$L = (x_2 - x_1) \text{ at } t_1 = t_2$$

- Length in its own frame $L'_0 = (x'_2 - x'_1) > L$, proof:

  $$L'_0 = \frac{(x_2 - v \times t_2)}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{(x_1 - v \times t_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  $$= \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} > L \quad \Rightarrow \quad L = L'_0 \times \sqrt{1 - \frac{v^2}{c^2}}$$
Time dilation

- Consider moving clock at $x' = \text{const}$ in its own frame
  $$\Delta t'_0 = (t'_2 - t'_1)$$

- Time between two events in our frame $\Delta t = (t_2 - t_1)$
  $$\Delta t = (t'_2 + \frac{v}{c^2}x')/\sqrt{1 - \frac{v^2}{c^2}} - (t'_1 + \frac{v}{c^2}x')/\sqrt{1 - \frac{v^2}{c^2}}$$
  $$\Delta t = \Delta t'_0/\sqrt{1 - \frac{v^2}{c^2}}$$
Length contraction and time dilation

- **Length contraction**
  
  When a body moves with speed $v$ relative to the observer, its length is contracted in the direction of motion by $\times \sqrt{1 - \frac{v^2}{c^2}}$

- **Time dilation**
  
  When a clock moves with speed $v$ relative to the observer, its rate is measured to have slowed down by $\times \sqrt{1 - \frac{v^2}{c^2}}$
Time dilation in particle physics

- Example: let's take cosmic ray muon $\mu^+$

  some facts: every minute 1 muon goes through 1 cm$^2$ area
  Quarknet: cosmic ray detectors, muon lifetime experiment

- Let's take typical muon energy 3 GeV
  - speed $= 0.9994 \times c$
  - lifetime $\Delta t'_0 = 2.2 \times 10^{-6}$ seconds
  - naive distance traveled (if there were no time dilation)
    $= 0.9994 \times c \times \Delta t'_0 = 659$ m
  - but $1/\sqrt{1 - \frac{v^2}{c^2}} = 29$
  - distance travel $\Delta L = 659 \text{m} \times 29 = 19000 \text{ m} = 19 \text{ km}$
Time dilation: muon

- Muon with $v = 0.9994 \times c$ and $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} = 29$

- From our point of view
  - muon is moving speed $v \simeq c$
  - time dilated
    \[ \Delta t = \Delta t'_0 \times \gamma \]
    - distance $\Delta L = 19$ km
    - time $\Delta t = 64 \times 10^{-6}$ s

- From muon point of view
  - Earth is moving speed $v \simeq c$
  - distance contracted
    \[ \Delta L'_0 = \Delta L / \gamma \]
    \[ \Delta L'_0 = 659 \text{ m} \]
  - time $\Delta t'_0 = 2.2 \times 10^{-6}$ s
“Time Travel”

- Cannot move “backwards in time”
- Move “forward” with different speed
- You can change “your clock”, but not direction:
  - biology: hybernate
  - physics: relativistic speed
Twin paradox

- If one could reach $v \simeq 0.9995 \times c$ (we will see why not)
  - time $\gamma = 1/\sqrt{1-v^2/c^2} \simeq 30$ times slower (like muon earlier)
  - get on a rocket and come back in 90 years
- twin on Earth 90 years older
- twin on rocket 3 years older
Conservation of Energy and Momentum?

- Energy and Momentum conserve, but different definition:

\[ E = mc^2 = m_0c^2/\sqrt{1 - \frac{v^2}{c^2}} \]

\[ p = mv = m_0v/\sqrt{1 - \frac{v^2}{c^2}} \]

- Impossible to reach speed of light \( v = c \iff E = \infty \) unless you are massless (\( m_0 = 0 \), like photon)

- Reverse is true: massless \( \Rightarrow v = c \) in all frames
  otherwise \( p = 0 \) and \( E = 0 \) \( \Rightarrow \) like nothing

- One can convert mass into energy and energy into mass
  relativistic mass is energy
Reaching Highest Energy

- $mc^2 = E$

$\nu/c = 0.999999991$
Large Hadron Collider: in Operation Now
Extra Dimensions of Space?

- Ideas of extra dimensions of space-time
  
  limit size of 5th dim $d_5 < 10^{-19}$ m

- Extra dimensions, Higgs, and other searches on LHC
What Gives Mass to Us: Proton, Neutron,...

- Remember Einstein’s formula

\[ E = mc^2 \]

\[ m(u \text{ or } d) < 1\% \ m(\text{proton}) \]

Mostly energy of gluons and quarks inside gives proton mass

not Higgs mechanism directly

but it is important \((m_d > m_u)\)