Condensed Matter Physics

Homework Assignment 13

Due date Friday, December 10

1. Landau theory for the Ising magnet. The Landau free energy density for an Ising magnet in external magnetic field h is

$$\mathcal{F}(m) = am^2/2 + bm^4/4 - hm$$

as a function of magnetization m. The coefficient of the quadratic term varies approximately linearly with temperature near the transition point T_c: $a = \alpha(T - T_c)$, where $\alpha > 0$; the quartic coefficient $b > 0$ can be regarded as a T-independent constant. In class we have shown that minimization of the free energy with respect to m in zero field yields $m = 0$ above the critical temperature; below it, the system spontaneously develops a magnetic moment $m = \pm\sqrt{\alpha(T_c - T)/b}$.

(a) Compute the magnetic moment $m(h)$ induced by the field above T_c. To do so, minimize the free energy $\mathcal{F}$ at a given h. (Hint: you may neglect the quartic term $bm^4/4$ above T_c for a weak field.)

(b) Show that the magnetic susceptibility

$$\chi(T) = \lim_{h\to 0} \frac{\partial m}{\partial h}$$

diverges on approach to the critical point T_c as a power of the reduced temperature: $\chi(T) = C/(T - T_c)^\gamma$. What is the value of the critical exponent γ?

2. Monte Carlo simulation of the Ising magnet. Jeff Wasserman has written a Java applet simulating an Ising magnet in two dimensions. You should be able to access it here: www.pha.jhu.edu/~javalab/ising/ising.html Make sure that Java is enabled in your browser.

(a) Familiarize yourself with the simulator. Enable data collection by switching to the *Intensive* regime. Sweep from low to high temperatures and observe the behavior of the specific heat C, magnetization M, and susceptibility χ. (If the plotted points lie on a horizontal line, clear the data and restart the sweep.) Most likely, the data will look noisy. It means that the system has not had enough time to equilibrate. To fix this problem, increase the measurement time (*Averages per Temp*). The problem is particularly severe near the transition temperature $T_c = 2.269$, especially in a large system. You might therefore want to switch to a smaller size.

(b) Once you have learned how to take data reliably, investigate the region of the phase transition. Using your favorite plotting program, or just pencil and graphing paper, plot the specific heat, magnetization, and susceptibility. The specific heat will show a logarithmic divergence $C \approx \text{const} - A \log |T - T_c|$, in contrast to a jump predicted by the Landau theory. Try to fit the data with a logarithmic function around T_c.

(c) The susceptibility will diverge as a power law, $\chi(T) \approx C_\pm/|T - T_c|^\gamma$; the constants C_+ and C_- refer to the temperatures above and below T_c. The exponent γ differs from the prediction of the Landau theory and is expected to be 7/4 in this numerical experiment. Plot the susceptibility versus $|T - T_c|$ on a logarithmic scale (both axes). Are your data points in agreement with the suggested power law? Determine the ratio of the critical amplitudes C_+/C_-.