

Five Dimensional Supersymmetric Gauge Theory

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When super-Yang-Mills field occupies 5D bulk and chiral fields live on 4D branes, supersymmetry breaking may be communicated from one boundary to another. In this paper, we investigate such a toy model. Using Kaluza-Klein expansion, we demonstrate a mechanism for generating SUSY breaking soft mass terms on the boundary.

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I. INTRODUCTION

Previously, a supersymmetric gauge theory, namely Mirabelli-Peskin model has been studied in both orbifold[1] and boundary picture[2]. In this model, ordinary matter field and supersymmetry breaking sector are located on two different 4 D branes, separated by a small distance in fifth dimension. This separation acts as a regulator, provides a possible mechanism of SUSY breaking mediation.

This model is inspired by Hořava and Witten's work[3], in which a 11-dimensional strong-coupling limit of Type IIA string theory was compactified on S^1/Z_2 , i.e. on an interval of length l bounded by mirror (orientifold) plane. Mirabelli-Peskin model is a further compactification, the main difference is that they used super Yang-Mills theory instead of supergravity as the bulk field.

Peskin and Mirabelli showed how to coupled 5D vector and hyper-multiplets to boundaries in a supersymmetric way, using an off-shell component formalism. However, the formalism is not familiar to 4D SUSY model-builders. In this paper, we will rephrase this 5 dimensional model in 4 dimensional superspace, then study SUSY breaking, try to establish a completed breaking spectrum by Kaluza-Klein expansion. In order to show the soft mass term generation, we will also discuss low energy theory.

The structure of this article is: in the first section, we present the Lagrangian of this theory. In section 2, we check the model's supersymmetry. Then a symmetry breaking term will be added in the section 3, the consequence spectrum and low energy theory will be discussed in the end.

II. SUMMARY OF MIRABELLI-PESKIN MODEL

The action of this model can be written as:

$$S = \int d^5x \left\{ \mathcal{L}_5 + \sum_i \delta(x^5 - x_i^5) \mathcal{L}_{4i} \right\}, \quad (1)$$

where the sum includes the walls at $x_i^5 = 0, l$.

The $N = 2$ five-dimensional supersymmetric theory can be described in terms of $N = 1$ four-dimensional superfields[4]. In the bulk, the vector supermultiplet (v_m, λ, D) and chiral

supermultiplet (ϕ_2, ψ_2, F_2) are described by a real vector superfield \mathbf{V} (in the WZ gauge) and a chiral superfield Φ_2 , respectively. On the brane, we use a chiral superfield Φ to describe the chiral multiplet (ϕ, ψ, F) . The supergauge transformation, parametrized by a chiral superfield Λ has the form:

$$\delta\mathbf{V} = \Lambda + \Lambda^\dagger, \quad \delta\Phi = -\Lambda\Phi, \quad \delta\Phi_2 = 2\partial_5\Lambda. \quad (2)$$

Let us consider a supergauge invariant Lagrangian that can be built from the superfields \mathbf{V} and Φ_2 ,

$$\mathcal{L}_5 = \frac{1}{4} \int d^2\theta \mathbf{W}^\alpha \mathbf{W}_\alpha + h.c. + \int d^2\theta d^2\bar{\theta} \mathbf{Z}^2, \quad (3)$$

where \mathbf{W} is the field strength for \mathbf{V} , the definition is

$$\mathbf{W}_\alpha = -\frac{1}{4} \bar{D} \bar{D} D_\alpha \mathbf{V} \quad (4)$$

and

$$\mathbf{Z} = \partial_5 \mathbf{V} - \frac{1}{2} (\Phi_2 + \Phi_2^\dagger) \quad (5)$$

Both \mathbf{W} and \mathbf{Z} are invariant under the supergauge transformation, Eq. (2).

The brane action can also be written in the superfield form as follow,

$$\mathcal{L}_4 = \int d^4\theta \Phi^\dagger e^{\mathbf{V}} \Phi. \quad (6)$$

In the orbifold picture, the (superfield) bulk-plus-brane Lagrangian is

$$\mathcal{L} = \mathcal{L}_5 + \mathcal{L}_4 \delta(x^5) = \frac{1}{4} \int d^2\theta \mathbf{W}^\alpha \mathbf{W}_\alpha + h.c. + \int d^4\theta \left\{ \mathbf{Z}^2 + \Phi^\dagger e^{\mathbf{V}} \Phi \delta(x^5) \right\}. \quad (7)$$

III. N=2 TO N=1 SUPERSYMMETRY BREAKING

It is clear that using the $D = 4, N = 1$ superfields keeps the $N = 1$ ($\theta = \theta_1$) supersymmetry manifest. A less obvious observation is that under the $N = 1$ supersymmetry, the bulk Lagrangian \mathcal{L}_5 varies into a total ∂_m (not ∂_M) derivative term, that is

$\delta_\theta \mathcal{L}_5$ does not contain a ∂_5 term.

(This is so because for the $D = 4, N = 1$ superfields x^5 is just a parameter. The highest component of a superfield varies into a total derivative, which is ∂_m for our superfields.)

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Another fact is that this model break the $N = 2$ supersymmetry. In order to show that, we write down the 2nd SUSY transformation (according to C. Xiong, personal communication, April 14, 2006):

$$\begin{cases} \delta_\eta \mathbf{V} = (\Phi_2 + \Phi_2^\dagger - 2\partial_5 \mathbf{V})(\theta\eta + \bar{\theta}\bar{\eta}) \\ \delta_\eta \Phi_2 = \eta_\alpha \mathbf{W}^\alpha \end{cases} \quad (8)$$

Under this transformation, the bulk Lagrangian becomes:

$$\begin{aligned} \mathcal{L}_5 \rightarrow \mathcal{L}_5 - \partial_5 \int d^4\theta \mathbf{Z}^2(\theta\eta + \bar{\theta}\bar{\eta}) \\ - \partial_5 \int \frac{1}{2} d^2\theta \mathbf{W}^\alpha \mathbf{W}_\alpha(\theta\eta + \bar{\theta}\bar{\eta}) . \end{aligned} \quad (9)$$

The result only differs from Eq. (3) by a total derivative, therefore they both lead to the same bulk action. It looks like the system still preserves $N=2$ supersymmetry.

However, the truth is, this transformation does not even exist. Orbitfold picture requires odd fields become zero on the brane. According to Eq. (8), the transformation parameter η must be odd, thus it must be zero. $N = 2$ supersymmetry then break into $N = 1$. This breaking is not spontaneous, but an explicit one due to geometry.

This fact is more explicit in the boundary picture, in which fields are not continuous on the branes anymore. Eq. (9) requires a translation symmetry to keep action invariant. However branes break it.

IV. WALL TO WALL SUPERSYMMETRY BREAKING

We have now described an explicit form for the coupling of 4-dimensional superfield on the brane to gauge superfield in the bulk. This formalism can be used to see how supersymmetry breaking on one wall is communicated to the other wall to provide soft supersymmetry-breaking terms. In this section, we consider a simplest example in which the wall at $x^5 = l$ contains no boundary matter fields. The gauge group is chosen to be $U(1)$, and a Fayet-Iliopoulos D term is added onto $x^5 = 0$ brane.

The full Lagrangian now becomes:

$$\begin{aligned} \mathcal{L} = \frac{1}{4} \int d^2\theta \mathbf{W}^\alpha \mathbf{W}_\alpha + h.c. \\ + \int d^4\theta \left\{ \mathbf{Z}^2 + \xi \cdot V \delta(x^5 - l) + \Phi^\dagger e^V \Phi \delta(x^5) \right\} \end{aligned} \quad (10)$$

Write down all x^5 sensitive term in the Equation of Motion, we have:

$$\partial_5 \left[2\partial_5 \mathbf{V} - (\Phi_2 + \Phi_2^\dagger) \right] = \xi \cdot \delta(x^5 - l) + \Phi^\dagger e^V \Phi \cdot \delta(x^5) \quad (11)$$

Using boundary picture, we require jump conditions on branes as:

$$\left[2\partial_5 \mathbf{V} - (\Phi_2 + \Phi_2^\dagger) \right]_l = \xi \quad (12)$$

$$\left[2\partial_5 \mathbf{V} - (\Phi_2 + \Phi_2^\dagger) \right]_0 = \Phi^\dagger e^V \Phi \quad (13)$$

Applying Z_2 symmetry, we have

$$2\partial_5 \mathbf{V} - (\Phi_2 + \Phi_2^\dagger)_{l-} = -\xi/2 \quad (14)$$

$$2\partial_5 \mathbf{V} - (\Phi_2 + \Phi_2^\dagger)_{0+} = \frac{1}{2} \Phi^\dagger e^V \Phi \quad (15)$$

In order to get complete solution, it is better to write EOM in component field form, the detail of component calculation has been included in the Appendix, only results are presented here.

In the bulk, super Yang-Mills multiplet satisfy these equations of motion:

$$\partial_M \partial^M (\phi_2 + \phi_2^*) = 0 \quad (16)$$

$$\partial_m \partial^m (\phi_2 - \phi_2^*) - 2i\partial_5 \partial_m v^m = 0 \quad (17)$$

$$-\partial_5 \partial_5 v^m + \partial_n v^{nm} - \frac{i}{2} \partial_5 (\partial^m \phi_2 - \partial^m \phi_2^*) = 0 \quad (18)$$

$$\sigma^m \partial_m \bar{\psi}_2 - \sqrt{2} \partial_5 \lambda = 0 \quad (19)$$

$$\sqrt{2} \sigma^m \partial_m \bar{\lambda} + \partial_5 \psi_2 = 0 \quad (20)$$

where index M runs 0,1,2,3,5.

Eq. 16 is an ordinary Klein-Gordon equation in 5D. When we later expand it in Kaluza-Klein modes, the fifth dimension dynamics will bring a mass term into 4D effective theory.

Define another ‘‘odd’’ vector field as

$$g_m = 2\partial_5 v_m + i\partial_m (\phi_2 - \phi_2^*) \quad (21)$$

Eq. 17 and 18 can be simplified as

$$\partial_n v^{nm} = \frac{1}{2} \partial_5 g^m \quad (22)$$

$$\partial_m g^m = 0 \quad (23)$$

According to our brane setup, g -field satisfies boundary conditions:

$$g^m|_{x^5=0} = j^m \quad (24)$$

$$g^m|_{x^5=l} = 0 \quad (25)$$

where current j_m on the physics brane is

$$j_m = \frac{i}{2} (\phi \mathcal{D}_m \phi^* - \phi^* \mathcal{D}_m \phi) + \frac{1}{2} \psi \sigma_m \bar{\psi} \quad (26)$$

Thus, for v -field, on each 4D slice, we have ordinary Maxwell equation with source.

When Weyl spinors ψ_2 and $\sqrt{2} \cdot \bar{\lambda}$ form a Dirac spinor, Eq. 19 and 20 will unite a Dirac equation. Kaluza-Klein expansion turns ∂_5 terms into mass terms, thus we will have a massive Dirac equation.

V. SPECTRUM

Let $\phi_2 + \phi_2^* = \varphi$, we can reduce the previous Eq. 16 and boundary conditions as

$$\begin{cases} (\square - \partial_5 \partial_5) \varphi = 0 \\ \varphi|_0 = -\frac{1}{2} \phi \phi^* \\ \varphi|_l = \frac{\xi}{2} \end{cases} \quad (27)$$

Even though φ is “odd” in the covering space, when we only care about expansion inside the orbitfold, there are two ways to extend bulk to get periodic $2l$ condition. φ has two forms of Kaluza-Klein expansion:

$$\varphi(x) = \sum_N u_N^{(+)}(\bar{x}) \cdot \cos\left(\frac{N\pi}{l}x^5\right) \cdot \epsilon_M(x) \quad (28)$$

and

$$\varphi(x) = \sum_N u_N^{(-)}(\bar{x}) \cdot \sin\left(\frac{N\pi}{l}x^5\right). \quad (29)$$

where multi-step function is defined by ordinary step function $\epsilon(x)$

$$\epsilon_M(x) = \begin{cases} \epsilon(x), & \text{if } 2N \cdot l \leq x < (2N+1) \cdot l \\ -\epsilon(x), & \text{if } (2N-1) \cdot l \leq x < 2N \cdot l \end{cases} \quad (30)$$

$u_N^{(\pm)}(\bar{x})$ are solutions of 4D massive Klein-Gordon equation:

$$\square + m^2 u_N(\bar{x}) = 0 \quad (31)$$

Obviously, the scalar field has 4D effective mass

$$m_N^2 = \left(\frac{N\pi}{l}\right)^2. \quad (32)$$

For the low energy case, we can think l effectively very small, thus Kaluza-Klein tower has large separation between modes, it is plausible to keep only leading order in Eq. 28 and 29:

$$\varphi(x) = u_0^{(+)}(\bar{x}) \cdot \epsilon_M(x) + u_1^{(+)}(\bar{x}) \cdot \cos\left(\frac{\pi}{l}x^5\right) \cdot \epsilon_M(x). \quad (33)$$

and

$$\varphi(x) = u_1^{(-)}(\bar{x}) \cdot \sin\left(\frac{\pi}{l}x^5\right) \quad (34)$$

The equivalence between Eq. 28 and Eq. 29 requires in the leading order:

$$u_1^{(-)} = \frac{4}{\pi} u_0^{(+)} \quad (35)$$

Actually,

$$\begin{aligned} u_1^{(-)} &= \frac{4}{\pi} u_0^{(+)} - \sum_{n=1}^{\infty} \frac{4}{\pi} \frac{1}{4n^2 - 1} u_{2n}^{(+)} \\ &= \frac{4}{\pi} u_0 - \frac{1}{3} u_2 - \frac{1}{15} u_4 - \dots \end{aligned} \quad (36)$$

In order to fulfill boundary conditions of Eq. 27, u must satisfy

$$\begin{cases} u_0^{(+)} + u_1^{(+)} = -\frac{1}{2} \phi \phi^* \\ u_0^{(+)} - u_1^{(+)} = \frac{\xi}{2} \end{cases} \quad (37)$$

Thus on the brane $x^5 = 0$, the auxiliary field D is:

$$D = \frac{1}{2} \partial_5 \varphi|_0 = \frac{\pi}{2l} \cdot u_1^{(-)} = \frac{2}{l} \cdot u_0^{(+)} = \frac{\xi + \phi \phi^*}{2l}. \quad (38)$$

Now, plug Eq. 38 into Eq. 44, we get 4D effective Lagrangian:

$$\mathcal{L}_4 = \mathcal{L}_{4, \text{massless}} + \frac{\xi}{2l} \frac{\phi \phi^*}{2} + \text{higher order term} \quad (39)$$

Thus susy breaking on another brane brings a soft mass term on the physical brane.

Fayet-Iliopoulos mechanism sometimes can be compensated by shifting the vacuum expectation values of a scalar field. Then there will be no soft mass term in tree level. In case of that, we need go to 2-loop contribution to study susy breaking.

VI. CONCLUSION

In this work we have shown that how susy breaking located on a 4D brane can communicate to another 4D brane where ordinary matter field is located. Some phenomenology model can be built on this mechanism. This project is not completed, consequent work includes Kaluza-Klein expansion with different boundary condition and solution of equation of motion in superfield language. Although here we only describe U(1) gauge in D-term breaking case, similar discussion can be extended to non-abelian gauge. Furthermore, the understanding of Yang-Mills theory may shine a light to the study of more realistic models, such as supergravity.

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Appendix

The vector and chiral superfields have the following component expansions,

$$\mathbf{V} = -i\theta\sigma^m\bar{\theta}v_m + i\theta^2\bar{\theta}\lambda - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D \quad (40)$$

$$\begin{aligned} \Phi &= \phi + i\theta\sigma^m\bar{\theta}\partial_m\phi + \frac{1}{4}\theta^2\bar{\theta}^2\partial_m\partial^m\phi \\ &\quad + \sqrt{2}\theta\psi + \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\sigma^m\partial_m\psi + \theta^2 F. \end{aligned} \quad (41)$$

The chiral superfields can be more conveniently written in terms of the “ y coordinates” ($y^m = x^m + \theta\sigma^m\bar{\theta}$),

$$\Phi(y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y). \quad (42)$$

We write down the bulk and brane Lagrangian in terms of these fields as:

$$\begin{aligned} \mathcal{L}_5 &= -\frac{1}{4}v_{mn}v^{mn} - \frac{1}{2}(\partial_5 v_m)(\partial_5 v^m) - \frac{1}{2}\partial_m\phi_2\partial^m\phi_2^* + \frac{1}{2}F_2 F_2^* + \frac{1}{2}D^2 \\ &\quad - \frac{i}{2}(\partial_m\phi_2 - \partial_m\phi_2^*)\partial_5 v^m + \frac{1}{2}D\partial_5(\phi_2 + \phi_2^*) \\ &\quad - \frac{i}{2}\lambda\sigma^m\partial_m\bar{\lambda} + \frac{i}{4}\psi_2\sigma^m\partial_m\bar{\psi}_2 + \frac{i}{2\sqrt{2}}(\psi_2\partial_5\lambda - \lambda\partial_5\psi_2) + h.c. \end{aligned} \quad (43)$$

$$\begin{aligned} \mathcal{L}_4|_{x^5=0} &= -\mathcal{D}_m\phi\mathcal{D}^m\phi^* - i\bar{\psi}\bar{\sigma}^m\mathcal{D}_m\psi + FF^* \\ &\quad + \frac{i}{\sqrt{2}}(\phi^*\lambda\psi - \phi\bar{\lambda}\bar{\psi}) + \frac{1}{2}\phi\phi^* D. \end{aligned} \quad (44)$$

where $\mathcal{D}_m = \partial_m + \frac{i}{2}v_m$.

and

$$\mathcal{L}_4|_{x^5=l} = \frac{1}{2}\xi D. \quad (45)$$

Equations of motion can be derived[5] by varying Eq. 43

$$-\partial_5\partial_5 v^m + \partial_n v^{nm} - \frac{i}{2}\partial_5(\partial^m\phi_2 - \partial^m\phi_2^*) = 0 \quad (46)$$

$$D + \frac{1}{2}\partial_5(\phi_2 + \phi_2^*) = 0 \quad (47)$$

$$-\partial_m\partial^m\phi_2 + i\partial_5\partial_m v^m + \partial_5 D = 0 \quad (48)$$

$$\sigma^m\partial_m\bar{\psi}_2 - \sqrt{2}\partial_5\lambda = 0 \quad (49)$$

$$\sqrt{2}\sigma^m\partial_m\bar{\lambda} + \partial_5\psi_2 = 0 \quad (50)$$

$$F_2 = 0 \quad (51)$$

Using the component expansions, we can split this superfield boundary condition into the following relations for the component fields,

$$\begin{aligned}
1 & : & -(\phi_2 + \phi_2^*) & \stackrel{0\pm}{=} & \frac{1}{2}\phi\phi^* \\
\theta & : & -\sqrt{2}\psi_2 & \stackrel{0\pm}{=} & \frac{1}{\sqrt{2}}\phi^*\psi \\
\theta^2 & : & -F_2 & \stackrel{0\pm}{=} & \frac{1}{2}\phi^*F \\
\theta\sigma^m\bar{\theta} & : & -2\partial_5 v_m - i(\partial_m\phi_2 - \partial_m\phi_2^*) & \stackrel{0\pm}{=} & -\frac{i}{2}(\phi\mathcal{D}_m\phi^* - \phi^*\mathcal{D}_m\phi) \\
& & & & -\frac{1}{2}\psi\sigma_m\bar{\psi} \\
\bar{\theta}^2\theta & : & -2i\partial_5\lambda - \frac{i}{\sqrt{2}}\sigma^m\partial_m\bar{\psi}_2 & \stackrel{0\pm}{=} & -\frac{1}{2}\lambda\phi\phi^* + \frac{1}{\sqrt{2}}\psi F^* \\
& & & & + \frac{i}{2\sqrt{2}}\sigma^m(\phi\mathcal{D}_m\bar{\psi} - \bar{\psi}\mathcal{D}_m\phi) \\
\theta^2\bar{\theta}^2 & : & \partial_5 D - \frac{1}{4}\partial_m\partial^m(\phi_2 + \phi_2^*) & \stackrel{0\pm}{=} & \frac{1}{2}\mathcal{L}_4^r + \frac{1}{8}\partial_m\partial^m(\phi\phi^*) .
\end{aligned} \tag{52}$$

where

$$\begin{aligned}
\mathcal{L}_4^r & = -\mathcal{D}_m\phi\mathcal{D}^m\phi^* + FF^* + \frac{1}{2}\phi\phi^*D \\
& - \frac{i}{2}(\psi\sigma^m\mathcal{D}_m\bar{\psi} - \mathcal{D}_m\psi\sigma^m\bar{\psi}) + \frac{i}{\sqrt{2}}(\phi^*\lambda\psi - \phi\lambda\bar{\psi}) . \tag{53}
\end{aligned}$$

and,

$$\begin{aligned}
1 & : & -(\phi_2 + \phi_2^*) & \stackrel{l_-}{=} & -\frac{1}{2}\xi \\
\theta & : & -\sqrt{2}\psi_2 & \stackrel{l_-}{=} & 0 \\
\theta^2 & : & -F_2 & \stackrel{l_-}{=} & 0 \\
\theta\sigma^m\bar{\theta} & : & -2\partial_5 v_m - i(\partial_m\phi_2 - \partial_m\phi_2^*) & \stackrel{l_-}{=} & 0 \\
\bar{\theta}^2\theta & : & -2i\partial_5\lambda - \frac{i}{\sqrt{2}}\sigma^m\partial_m\bar{\psi}_2 & \stackrel{l_-}{=} & 0 \\
\theta^2\bar{\theta}^2 & : & \partial_5 D - \frac{1}{4}\partial_m\partial^m(\phi_2 + \phi_2^*) & \stackrel{l_-}{=} & 0 .
\end{aligned} \tag{54}$$

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