Cosmic Rays, Fermi acceleration, Ultra-High-Energy Cosmic Rays (UHECRs), and Ultra-High-Energy (UHE) Neutrinos

Everywhere that there are astrophysical plasmas there are cosmic rays which I here define to be high-energy particles (e.g., protons and electrons) with non-thermal power-law spectra,

\[ \frac{dN}{dE} \propto E^{-\gamma} \quad \text{with} \quad \gamma \approx 2.2-2.7 \quad \text{typically}. \]

E.g., CRs are inferred through the synchrotron emission they emit in SNRs, radio lobes, AGN jets, the Milky Way -- GRBs, etc. cluster shocks,...

CR protons are seen in the Milky Way; they are also rarely seen in the solar wind.

Thermal processes produce thermal energy distributions which are exponentially (Boltzmann) suppressed, \( \exp[-E/kT] \) at high energies; the power laws must therefore be non-thermal:

\[
\begin{align*}
\log N & \quad \text{as thermal} \\
\log E & \quad \text{as CR}
\end{align*}
\]

The basic idea about how CRs are accelerated is due to Fermi. The detailed implementation in any given system may be complicated but the basic idea is easily illustrated with a toy model.

Suppose a particle with velocity \( V \) is incident on a cloud of plasma, with \( B \) fields out of the page, moving toward the particle with velocity \( V \).
The magnetized cloud acts like a moving mirror that reflects the particle. Since it is moving, the particle receives an energy kick, much like a ping-pong ball when you hit it with a paddle.

If the reflection is elastic in the mirror frame, then the reflected particle has energy

\[ E_{\text{out}} = \gamma^2 \left( \frac{1 + 2 \frac{V}{c} \frac{V}{c} + \frac{V^2}{c^2}}{1 - \frac{V^2}{c^2}} \right) E_{\text{in}}, \]

where \( \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \).

Thus, if the particle is relativistic (\( \frac{V}{c} \approx 1 \)), then

\[ E_{\text{out}} = \left( 1 + 2 \frac{V}{c} + \frac{V^2}{c^2} \right) \gamma^2 E_{\text{in}} \]

(which recovers \( E_{\text{out}} = 4 \gamma^2 E_{\text{in}} \) familiar from inverse Compton scattering by a relativistic \( \gamma \) in the \( \gamma \gg 1 \) limit and \( 1 + 3 \frac{V}{c} \) in the NR limit).

Now suppose that there are two clouds approaching with velocities \( \pm V \):

![Diagram of two clouds approaching with velocities ±V](image)

The particle can then keep bouncing off the clouds, increasing its energy by a multiplicative factor,

\[ P = \frac{4 \gamma^2 V^2 + \gamma^2 (1 + \frac{V}{c})^2 \gamma^2}{\gamma^2} \text{ each time. After bouncing } n \text{ times, it will have an energy} \]

\[ E(n) = P^n E_{\text{in}} \]

Suppose that after each scatter, there is a chance \( f \) that the particle escapes the system. If so, then the number of particles with that scatter \( n \) times (to an energy \( E(n) \)) before escaping is

\[ N(n) \propto (1 - f)^n \]
Writing \( N = \frac{\frac{\hbar}{\ln P}}{\ln P} \), and with some algebraic rearrangement, the number of particles with energy \( E \) is

\[
N(E) \propto E^{\gamma_1 - \gamma} \quad \text{with} \quad \gamma_1 - \gamma = \frac{\ln P \ln (1 - \gamma)}{\ln P}
\]

or

\[
\frac{dN}{dE} \propto E^{\gamma - \gamma_1}
\]

and this is how a power-law spectrum arises. A little more work shows (e.g., see Langer for MR case) why \( \gamma = 2 - 2.5 \) may be expected.

More realistically, these "clouds" may be magnetic-field irregularities or (more likely) the magnetized fluid in front and behind a shock.

Conceptual question: How do such large non-thermal energies consistent with equipartition? Answers CR acceleration can be viewed as approach to equipartition between particles \( m_0 \) (mass m) and \( B \) the fluid motions of bulk flows of fluids with macroscopic masses \( M \gg m \). Even if \( V_{\text{bulk}} \ll c \), will have \( MV^2 \gg mc^2 \).

Local CR flux (contains \( e^- \), \( e^+ \)'s, \( p \)'s, heavier elements, but most of energy density is in protons)

\[
\text{Flux} \quad \left( \text{m}^2 \text{sec}^{-1} \text{sr}^{-1} \text{GeV}^{-1} \right) \quad 10^{-12} \quad 10^{-13} \quad 10^{-14} \quad 10^{-15} \quad 10^{-16}
\]

\[
\text{E (eV)} \quad 10^9 \quad 10^{10} \quad 10^{11} \quad 10^{12} \quad 10^{13} \quad 10^{14} \quad 10^{15} \quad 10^{16}
\]

\[
\text{extragalactic} \quad \text{MW} \quad \text{knee} \quad \text{ankle''} \quad \text{UHECRs (ULTRACRS)}
\]

affected by solar \( \delta \)-fields
One more fact about accelerations:

The maximum energy to which a CR can be accelerated is limited by the (Larmor radius) or size of accelerating region:

\[ R_L = \frac{xMv}{eB} \approx \frac{E}{eB} \]

\[ \Rightarrow E_{\text{max}} \approx 10^{21} \text{eV} \left( \frac{B}{G} \right) \left( \frac{R}{\text{pc}} \right) \]

E.g., a SNR \( (R \sim 10 \text{pc}, B \sim 10^{-6} \mu G) \) \( \Rightarrow E_{\text{max}} \approx 10^{15} \text{eV} \)

\( \Rightarrow \) sub-ankle CRs thought to be accelerated in SNRs.

And MW \( (\sim 10 \text{ kpc}, \mu G) \) can contain CRs up to \( \sim 10^{18} \text{eV} \) \( \Rightarrow E \sim 10^{18} \text{eV} \) are extragalactic.

Can put potential accelerators on Hillas plot:

![Hillas plot diagram](image)
UHECRs \((E \gtrsim 10^{19}\text{ eV} - 10^{20}\text{ eV})\) extragalactic:

If CR proton has energy above

\[
E \simeq \frac{m_n^2}{E_{\text{GZK}}} \simeq \frac{(140 \text{ MeV})^2}{3 \times 10^4 \text{ eV}} \approx 10^{20} \text{ eV}
\]

(We GZK—Greisen-Zatsepin-Kuzmin) bound, it \(n + \pi^+\)
can produce a pion through \(p + \pi^- \rightarrow n + \pi^+\).

With the mean-free-path for this is

\[
\lambda = \frac{1}{n_f} \approx \frac{1}{(411 \text{ cm}^3) \cdot (4.2 \times 10^{-5} \text{ cm})^2} \approx \frac{1}{(3 \times 10^{-16} \text{ cm})}
\]

\(\sim 100 \text{ Mpc}\).

Thus any sources producing \(\gtrsim 10^{20} \text{ eV}\) particles
that we see must be within \(\sim 100 \text{ Mpc}\) distance.

\(\Rightarrow\) Expect decline in flux at \(\gtrsim 10^{20} \text{ eV}\);
is now seen by Auger, a 3000-km\(^2\)
ground array in Argentina (Nov. 2007).

Energy density in \(\gtrsim 10^{19} \text{ eV} \) CRs is \(\sim 5 \times 10^{52} \frac{\text{erg}}{\text{Mpc}^3}\)

Energy production rate \(\sim 5 \times 10^4 \text{ erg/Mpc}^3/\text{yr}\)

Waxman-Bahcall limit to UHE neutrinos:

If UHECR protons lose energy in source and through
\(x p \rightarrow n + \pi^+\) or \(pp \rightarrow p + p + \pi^+ + \pi^+ + \ldots\)
chained pions decay to \(\nu s\): \(\pi^- \rightarrow \mu^- + \nu_e, \mu^+ + \nu_e, e^- + \nu_e, \nu_e\).
The maximum intensity of such \(\nu s\) is

\[I_{\text{max}} \approx 10^{-8} \text{ GeV/cm}^2/\text{s/str} \quad E_{\nu} \sim E_{\text{p}}/20\]

Constrains models where \(\pi^-\nu\) produces large gamma-ray background.