Gamma-Ray Bursts:

(source: notes provided by R. Sari)

Are very bright bursts (e.g., 10^6 erg/cm^2) that last ~2 sec — 100s of secs (are also some with durations 0.01s ≤ t ≤ 2.5, but these are now thought to be something else). Light curves highly variable and often irregularly so. Variability has been observed on timescales t = m sec. No two look alike.

Spectra:

Again, are highly irregular and vary over course of the burst. Still, often parametrized in terms of a parameter "Band function" (Band 1993):

\[ N(E) \propto \frac{E}{A E_0^{\beta}} \left( \frac{E}{E_0} \right)^{\gamma} \left( 1 + \left( \frac{E}{E_0} \right)^{\alpha} \right)^{-1} \]

\( E_0 \) typical values ~100 keV ≤ \( E_0 \) ≤ 400 keV \( \alpha = -1 \)
\( \beta = 2.25 \)

(i.e., two power laws \( \propto E^{-\gamma} \) at low \( E \) and \( E \propto E^{\beta} \) at high \( E \))

High-energy power-law extends as high as can be seen (e.g., 200 MeV for GRB 930506, ~10^3 peak energy)

\( \Rightarrow \) Highly non-thermal \( \Rightarrow \) Source must be optically thin!
Rate: \( \sim 200 \text{ yr} \) at \( f \geq 10^{-7} \text{ erg/cm}^2/\text{sec} \)

Distribution: constant with isotropic

\( \Rightarrow \text{cosmological; have redshifts} \quad 0 < z \leq 8.4 \)

Energy: If distance is \( D \sim 3 \text{ Gpc} \)

\[ E = \frac{3 \times 10^5 \text{ erg/cm}^2}{(1+z)} \quad \text{(if energy radiated isotropically)} \]

\( \text{e.g., for } F = 10^{-6} \text{ erg/cm}^2 \text{ as high as } 10^{5.5} \text{ erg} \)

Compactness Problem:

A variability timescale \( \Delta t \) implies a source size \( R \ll c \Delta t \). If the energy in \( 10^5 \text{ s} \) of \( 10^{20} \text{ MeV} \) is \( E \), the optical depth, as a lower-energy \( \delta \) passes through the source to produce \( e^+ e^- \) via \( \delta + \gamma \rightarrow e^+ e^- \), is

\[ \tau = \frac{N}{\Delta t} \frac{4 \pi R^2}{E} = 3 \times 10^7 \left( \frac{\Delta t}{0.1 \text{ sec}} \right) \left( \frac{E}{10^{51} \text{ erg}} \right) \gg 1 \]

(Ruderman 1974).

Would therefore produce \( e^+ e^- \) pairs which would then dominate the x-ray, conflicts with power-law N(E).

Relativistic Outflows:

Solve compactness problem if emitting regions move toward us at Lorentz factor \( \gamma \gg 1 \).

First of all, \( \Delta t_{\text{obs}} = \frac{\Delta t_{\text{em}}}{\gamma^2} \) \((\gamma \gg 1)\) for AGN jets; this gives factor \( 2 \times 10^4 \).

Moreover, if observed brightness is \( S \), \( \frac{dN}{dE} \propto \frac{E^{-2}}{E_{\text{obs}}} \)

The observed energy is \( E_{\text{obs}} = \gamma E \)
Planck number

Therefore, number of $\gamma$'s with energy above $e\gamma - \text{threshold}$ in the emitter (moving) frame is smaller than naive estimate. This adds any additional $\gamma^{-2}$ for flat-spectrum sources.

$$(dN/E) \propto E^{-2}.\]

$\Rightarrow \quad \gamma \propto \gamma^{-6} \quad \Rightarrow \quad \gamma \ll 100$$

The Fireball Model:

1. **Source:** remains a mystery but must have enough energy, correct rate of occurrence, and compact. Finally, it must solve the baryon pollution problem.

Suppose we have total energy $E$ in small (in $s^4$ baryons $e\gamma^{-6}$) initially in small region of baryon rest-mass $M = N\mu$. The maximum achievable Lorentz factor is $\gamma_{\text{max}} = E/\mu c^2$.

A given Lorentz factor $\gamma$ requires a baryonic mass,

$$M \leq \frac{E}{\gamma c^2} = 6 \times 10^{-6} M_\odot \left(\frac{E}{10^{51} \text{erg}}\right)^2$$

Candidate sources include

1. Neutron-star mergers
2. Failed supernova
3. Collapse of magnetized WD
4. Explosive of mass $\lesssim 1$ solar mass

Associations of $SN1998bw \leftrightarrow GRB 980425$

and $SN2003dh \leftrightarrow GRB 030329$

$\Rightarrow$ GRBs (at least some) are some type of SN; perhaps of rotating star that collapses to BH + accretion disk.
Fireball model: general picture

An initially optically thick region of material expands under its own pressure until it gets \( \gamma = E/M \), whereupon it becomes optically thin.

**Evolution:**

Assume a relativistic expand shell of material of pressure \( p \), baryon rest mass density \( \rho \), width \( \Delta \) (observer frame), Lorentz factor \( \gamma \), and radius \( R \).

\[
8^3(4\pi/3 + pc^2)\Delta R^2 = \text{const} \tag{conservation of energy}
\]

\[
Yp\Delta R^2 = \text{const} \tag{cons. of baryons}
\]

\[
p(\gamma \Delta R^2)^{\gamma/3} = \text{const} \tag{cons. of entropy}
\]

assuming \( EoS \), \( e = 3\gamma \).

Solutions are an initial radiation-dominated phase with \( D = \text{const} \), \( Y \propto R \), \( p \propto R^3 \) followed by a matter-dominated phase with \( D = \text{const} \), \( Y = \text{const} \), \( p \propto R^2 \), until \( R/\Delta^2 \approx \Delta \), at which point the shell spreads in width with \( D = R/\Delta^2 \).

Thermalizing the energy:

The relativistic shell must run into something to thermalize the baryon and emit x-rays.

Two possibilities:

1. **External shocks**: the shell runs into the ISM

2. **Internal shocks**: flow is irregular and multiple inner shells collide with each other
External shocks have trouble producing variability, but may be responsible for late-time lower-energy afterglows.

Internal shocks require the source to be variable.

Suppose the outflow is composed of many relativistic shells with separation by \( S \) and let \( D \) be the distance between the first and last shell.

Suppose two adjacent shells have Lorentz factors \( \gamma \) and \( \gamma_* \). They collide at distance \( R = x^d \) and produce pulse of duration \( \Delta t = R/v^2c = \delta/c \).

Converting Thermal Energy to Radiation:

The co-shell collisions result in relativistic shocks and which produce strong acceleration \( e^{-5} \) and produce B-fields via plasma instabilities. The observed \( x \)-rays are then synchrotron radiation and inverse-Compton scattered synchrotron x-rays.

The typical frequency formula from an e\(^-\) or Lorentz factor \( \gamma_e \) is

\[
\nu = \frac{1}{2\pi} \frac{eB}{mc} \gamma_e^2
\]

The power of each e\(^-\) is

\[
P = \frac{4}{3} \nu (\gamma_e^2) \frac{B^2}{8\pi}
\]

**GRB Afterglows:**

The initial e\(^-\) in GRB is followed over long times (~months) by radiation at lower frequencies (x-ray, UV, optical, IR, radio), with diminishing intensity and frequency.

These afterglows are believed to be due to the external shock produced by the interaction of the overall relativistic outflow into the ISM, like a relativistic SNR.
Suppose the initial ISM has uniform proton density \( n \) and that the shock moves through it, at \( \alpha \). Behind the shock, the particle density is \( 4 \alpha n \) and the energy density is \( 4 \alpha \gamma n c^2 \), where \( \gamma \) is the Lorentz factor of the shocked fluid.

We assume that Fermi acceleration produces a power-law distribution of \( \delta \) Lorentz factors \( \gamma_e \) (in the fluid rest frame) of

\[
N(\gamma_e) \, d\gamma_e \propto \gamma_e^{-\delta} \, d\gamma_e \quad \text{for} \quad \gamma_e > \gamma_m,
\]

and we take \( \delta > 2 \). Assume a constant fraction \( \epsilon_\delta \) of the shock energy goes into \( \delta \). Then

\[
\gamma_m = 6 \epsilon_\delta \frac{\gamma_e}{\gamma_e + \gamma_0} \quad \text{for} \quad \gamma_e > \gamma_0
\]

We also assume that a fraction \( \epsilon_B \) of the shock energy goes into the \( B \) field:

\[
B = (32 \pi M_p \epsilon_B n) \, \gamma_e
\]

**Synchrotron Spectrum of Relativistic Shock:**

The observed power and characteristic frequency are from an \( \epsilon \) of the ISM rest-frame kinetic energy with bulk flow \( \gamma \) toward us as are

\[
P(\nu_e) = \frac{4}{3} \epsilon_\delta \frac{c \nu_e^2 \gamma_e^2}{\gamma_0} \quad \text{and} \quad \nu(\nu_e) = \gamma_e \nu_e^2 \frac{\gamma_0 B}{\gamma_0^2 M_p c}
\]

The spectral power \( P_\nu \) (erg/s/Hz) is \( P_\nu \propto \nu^{4/3} \)

for \( \nu < \nu(\nu_e) \) and exponentially suppressed for \( \nu > \nu(\nu_e) \). The power peaks at \( \nu(\nu_e) \) at

\[
P_{\nu, \text{max}} \propto \frac{P(\nu_e)}{\nu(\nu_e)} = \frac{M_p c^2 \gamma_0 B}{\frac{3}{\gamma_0} \gamma_0} \]
So far, we have assumed that the energy radiated by an $e^-$ is small compared with its energy, but this will not be valid above a critical $\chi_c$ set by the condition

$$\chi(\chi_c) = \frac{3m_e}{4\pi^2} = \frac{3Mc}{\kappa \beta \gamma c^2 t} = \frac{3m_e}{\kappa \beta \gamma c^2 t} \chi_c^3,$$

where $t$ is the post-GRB observer time.

Consider an $e^-$ with initial $\chi > \chi_c$. This $e^-$ cools to $\chi_c$ in time $t$. As it cools, $\chi \propto \chi_c^2$, while $E_e \propto \chi_c^2$, so the $P_e \propto \chi_c^{-1/2}$ in the range $\chi(\chi_c) = \chi_c < \chi < \chi(\chi_c)$. Therefore, an $e^-$ with $\chi > \chi_c$ initially radiates

$$P_e < \begin{cases} \chi_c^{-1/2} & \chi_c < \chi < \chi(\chi_c) \\ \kappa \beta \gamma c^2 t^{-1/2} e^{-\chi(\chi_c)} & \chi > \chi(\chi_c) \end{cases}$$

$P_{\chi,\text{max}}$ now occurs at $\chi = \chi_c$.

This description is valid for (1) $e^-$ injection is instantaneous and observations averaged over time $t$ or (2) $e^-$ injection is continuous over time $t$ and the observation is instantaneous $\Rightarrow$ GRBs.

So far, we have considered just one $e^-$. We now have to integrate over $N(\chi) < \chi_c^{-1}$.

If $\chi_m > \chi_c$ (fast cooling), all $e^-$'s cool to $\chi_c$, and observed power at $\chi_c$ is $N_e P_{\chi,\text{max}}$. Then

$$\text{flux} \quad F_\chi = \begin{cases} \kappa \beta \gamma c^2 \langle \chi \rangle_{\text{obs}} P_{\chi,\text{max}} & \chi_c < \chi < \chi_m \\ \kappa \beta \gamma c^2 \langle \chi \rangle_{\text{obs}} P_{\chi,\text{max}} & \chi > \chi_m \end{cases},$$

where $\chi_m = \chi(\chi_m)$ and $P_{\chi,\text{max}} = N_e \frac{P_{\chi,\text{max}}}{4\pi D^2}$.
If \( V_c > \sqrt{V_m} \), only \( \sqrt{V} \) with \( V > V_c \) can cool. (Slow cooling).

Then,

\[
F_v = \begin{cases} 
\frac{(V/V_m)^{3/2} F_{\text{max}}}{\left[ (V/V_m)^{1/2} - (1 - \sqrt{V/V_m}) \right]} & V < V_m \\
V_m & V_c < V < V_m \\
\sqrt{V} & V > V_c 
\end{cases}
\]

At even lower frequencies, \( F_v \propto \sqrt{V_m} \), the synchrotron radiation becomes self-absorbed and \( F_v \propto \sqrt{V} \).

Self-absorption occurs at low \( V \) because the source-frame specific intensity cannot be higher than blackbody,

\[
I_{\nu,\text{BB}} = \frac{2\nu_0^2}{c^2} m_e c^2 V_0 \quad \text{taking} \quad kT_\text{eff} = V_0 m_e c^2
\]

Since \( I_\nu / V^3 \) is invariant, observer sees maximum,

\[
I_{\nu,\text{BB}} = \frac{2\nu_0^2}{c^2} m_e c^2 V_0 r.
\]

Observed size of emitting area is \( \frac{R}{\nu} \) and its angular size is \( R/\Delta \theta \), so the observer flux is limited by

\[
F_{\nu,\text{BB}} = I_\nu \Delta \Omega = \frac{2\nu_0^2}{c^2} m_e c^2 V_0 r \left( \frac{R}{\Delta \theta} \right)^2.
\]
The afterglow light curves (temporal evolution of $F_{\nu}$ at some frequency $\nu$) are obtained by considering the evolution of $x_{e}$, $N_{e}P_{\gamma_{\text{max}}}$, etc.

The total # of $e$'s in the post-shock fluid is $N_{e} = \frac{4\pi R^{3}n_{i}}{3}$.

Assume that the shock evolution is adiabatic (i.e., that radiative losses are negligible). Then the shock energy $E$ is constant and $E \approx \frac{8}{3}R^{3}n_{i}m_{e}c^{2}$. Using $t = R/v_{s}$,

$$R(t) = \left(\frac{17E t}{4\pi n_{i}m_{e}c^{4}}\right)^{1/3}$$
$$V(t) = \left(\frac{17E t}{108\pi n_{i}m_{e}c^{5}t^{3/2}}\right)^{1/3}$$

If so, then

$$V_{c} = 2.7 \times 10^{-2} \frac{E_{52}}{E_{52}} \frac{n_{i}}{10^{15} \text{ cm}^{-3}} \frac{t_{d}}{1 \text{ day}} \text{ Hz}$$

$$V_{m} = 5.7 \times 10^{-14} \frac{E_{52}}{E_{52}} \frac{E_{52}}{E_{52}} \frac{E_{52}}{E_{52}} \frac{t_{d}}{1 \text{ day}} \text{ Hz}$$

$$E_{\text{V_{max}}} = 1.1 \times 10^{52} \frac{E_{52}}{E_{52}} \frac{n_{i}}{10^{15} \text{ cm}^{-3}} \frac{D_{28}}{28} \mu J$$

where $t_{d} = (4/\text{day})$ (E_{52} = E/10^{52} ergs) $n_{i} = \frac{N_{i}}{10^{15} \text{ cm}^{-3}}$

and $D_{28} = D/10^{28} \text{ cm}$

The transition from fast to slow cooling occurs when $V_{c} = V_{m}$, which occurs at

$$t = 210 \frac{E_{52}}{E_{52}} \frac{E_{52}}{E_{52}} \frac{n_{i}}{10^{15} \text{ cm}^{-3}} \text{ days}$$

The time evolution of $V_{c}, V_{m}$, $E_{\text{V_{max}}}$ can give the light curve at a given frequency, $E_{\nu}$, at $V = 10^{-15} \text{ Hz} V_{5}$.

The frequencies $V_{c}, V_{m}$ equal $V$ when

$$t_{c} = 7.3 \times 10^{-6} \frac{E_{52}}{E_{52}} \frac{E_{52}}{E_{52}} \frac{n_{i}}{10^{15} \text{ cm}^{-3}} \frac{V_{15}^{2}}{V_{5}^{2}} \text{ days}$$

$$t_{m} = 0.69 \frac{E_{52}}{E_{52}} \frac{E_{52}}{E_{52}} \frac{n_{i}}{10^{15} \text{ cm}^{-3}} \frac{V_{15}^{2}}{V_{5}^{2}} \text{ days}$$
Since \( V_c = V_m \) at time \( t_0 \), we have either
\[ t_0 > t_m > t_c \quad \text{or} \quad t_0 < t_m < t_c. \]
Defining \( V_o = V_c(t_0) = V_m(t_0) \), we have
\[ V_o = 1.8 \times 10^{11} \text{ cm}^{-1} \text{ MeV} \text{ Hz}^{-1/2}. \]

For frequencies \( V > V_o \), \( t_0 > t_m > t_c \), this case is referred to as the high-frequency light curve.

For frequencies \( V < V_o \), \( t_0 < t_m < t_c \Rightarrow \) low-frequency light curve

The time-frequency dependence of the afterglow is written as
\[ E_V \sim V \tau. \]

If \( \beta = \frac{1}{2} \), as is case for both slow- and fast-cooling, then \( \alpha = 3(\beta - 1)/2 \).
If for some reason \( \beta = (\rho - 1)/2 \) (which is what you get for a non-cooling population of \( \epsilon_V \)), then \( \alpha = (3 - \rho)/2 \).

Jets and GRB energetics:

GRB energies implied by assuming isotropic emission get up to \( \approx 5 \times 10^{54} \text{ erg} \Rightarrow 10^{54} \text{ erg} \text{ cm}^2 \), which is kind of big.

It is now generally accepted that typical GRBs are beamed into a solid angle \( \approx 4\pi/100 \), and that the energies are \( \times 100 \times \) smaller.

The evidence comes from breaks in the afterglow power laws. At early times, the evolution is that of a jet. At some time \( t' \), when the shock has slowed, it begins to spread. The subsequent evolution is thus different, more appropriate for spherical rather than jet expansion.