When WD's mass exceeds \( M_{\text{crit}} \), it begins to collapse and p? At some point, the Fermi energy \( E_F = \frac{1}{2}(p^2 + m^2) \) exceeds \( (m_m - m_p)c^2 = 129 \text{ MeV} \). This happens at density \( \rho = 1.2 \times 10^7 \text{ g/cm}^3 \). At higher \( \rho \), it becomes energetically favorable for \( e^- \)s to inverse-beta decay (combine) with protons to form neutrons \( \Rightarrow \) neutron stars.

More realistically, WD contains CO, not free protons. What really happens is that \( e^- \)'s capture on nuclei to form increasingly \( n \)-rich nuclei. At density \( \rho = 1.5 \times 10^9 \text{ g/cm}^3 \), neutrons begin to bleed from nuclei. This "ionization" of \( n \)'s from nuclei softens the EOS, and star continues to collapse until \( \rho = 10^{16} \text{ g/cm}^3 \) at which point neutron degeneracy pressure becomes sufficient to support the star.

The density \( \rho_{\text{crit}} = 2 \times 10^9 \text{ g/cm}^3 \) is the density of ordinary nuclear matter. The EOS at higher densities is determined by nuclear theory, which is highly uncertain.

More quantitatively, there will be degenerate \( n, p \), and \( e^- \) with Fermi energies \( E_F(n), E_F(p), E_F(e^-) \). If
\[
E_F(n) < E_F(p) + E_F(e^-)
\]
then neutrons cannot \( \beta \) decay. In thermal equilibrium, \( E_F = E_F(n) + E_F(p) \), and
\[
\rho_F = \left( \frac{3\pi^2}{2} \right) \frac{n^3}{8}\pi
\]
for all. At \( \rho_{\text{crit}} \),
\[
E_F(n) = m_n c^2 + \frac{E_F(n)}{2m_n}, \quad E_F(p) = m_p c^2 + \frac{E_F(p)}{2m_p},
\]
but \( E_F(e^-) \approx 0 \).

Using \( \rho = 2 \rho_{\text{crit}} \), we find
\[
\left( \frac{3\pi}{8\pi} \right)^{1/3} \frac{\rho_{\text{crit}}}{2} + \left( \frac{3\pi}{8\pi} \right)^{1/3} \frac{129}{2m} - \left( \frac{3\pi}{8\pi} \right)^{1/3} \frac{129}{2m} = (m_m - m_p)c^2
\]
\[= 1.3 \text{ MeV} \]
Can then solve this at any density noting that
\[
(n_n + n_p) = \rho_{\text{crit}},
\]
E.g., at \( \rho_{\text{crit}} \), \( (n_e/n_n) \approx \frac{1}{200} \Rightarrow \text{ neutron star} ! \)
Mass-Radius equation for NSs:

The WD central density relation,

\[ P_c = \frac{3 \sqrt{2} \left( M^2 \right) \rho_n^3}{\left( m_n c^2 \right)^3} \]

is easily modified for NSs by \( \rho_n = 1 \quad m_n = m_n \).

Likewise, the radius is (roughly)

\[ R = \frac{2^{8/3}}{5} \times 0.77 \times \frac{v_n^{8/5} (M^2 / M)}{c} \times \frac{1}{6} \times \frac{1}{m_n c} \]

\[ R = (0.0135) \times 2^{8/3} \times \frac{m_n^{-2/3}}{m_n} \times \frac{(M_\odot / M)^{8/5}}{c} \times \frac{1}{6} \times \frac{1}{m_n c} \]

\[ \approx 2.1 \times 10^5 \times R_0 \times (M / 1.4 M_\odot)^{8/5} \approx 1.5 \times 10^6 \text{ cm} \times (M / 1.4 M_\odot)^{8/5} \]

\[ \approx 15 \text{ km} \times (M / 1.4 M_\odot)^{8/5} \]

A few comments/caveats:

1. As will be seen below, the (Schwarzschild) radius of a 1.4\( M_\odot \) BH is \( R_s \approx 3 \) km. Therefore, the NS is in the strong-field regime of GR. Alternatively, the redshift from the surface of the neutron star is

\[ \frac{\Delta \lambda}{\lambda} \approx 0.2 \]

Thus, GR corrections to the Newtonian eqn. of hydrostatic equilibrium will give rise to \( \xi(20\%) \) correction.

This GR eqn. of hydrostatic equilibrium is known as the Tolman-Oppenheimer-Volkoff (TOV) equation.
(2) The approximation \( P \propto R^2 \) used to derive \( M \propto R^2 \) breaks down already for \( \sim 1.4 \text{M}_\odot \); in fact \( \rho_c \approx n_c \), thus, and so \( \rho_c \) a more accurate EOS must be used.

(3) In our simple calculation, \( P_c \approx 3.3 \times 10^{17} \text{g/cm}^3 \).\text{acc.}

Uncertainties in the nuclear EOS at these high densities give rise to additional uncertainties in the \( M \) \( R \) relationship.

E.g., nucleon-nucleon repulsion might soften EOS, but if new particles (e.g., \( \Xi^0 \), \( \eta \), pions, hyperons) are produced at high \( \rho \), this might soften the EOS.

(4) The gravitational binding energy \( E_b \) is
\[
\frac{E_b}{c^2} = \frac{GM^2}{R} \approx 0.1 \ (1.4 \text{M}_\odot).
\]

Thus, the mass of a NS is smaller than might expect from Newtonian calculation.

---

Maximum Mass for NSs:

Incidentally, people now try to determine the nuclear EOS by measuring the \( M \) \( R \) relation for NSs. E.g., \( GM/R \) can be obtained from redshifts of lines emitted from the NS surface. The surface gravity \( GM/R^2 \) can be measured through its effect on pressure broadening of lines. A combined measurement of \( M/R \) and \( M/R^2 \) can be used to obtain \( M \) and \( R \). Are difficult but ambitious people are trying.
Maximal Mass for WDs:

Nearly scaling the WD result, we obtain

$$M_{\text{ch}} \approx 6 M_{\odot} \quad \text{for NS.}$$

However, all complications above are important and generally tend (especially GR) to reduce $M_{\text{ch}}$. State-of-the-art calculations for different EOSs give $M_{\text{ch}} \approx 1.5-3 M_{\odot}$.

For fun, suppose we had a star consisting of incompressible matter of density $\rho_0$. In Newtonian gravity,

$$\frac{dP}{dr} = \frac{-G M_{\text{ch}} \rho_0}{r^2} \Rightarrow P(r) = \frac{2\pi}{3} G \rho_0 R^3 (R^2 - r^2), \quad \text{and}$$

$$P_c = P(r=0) = \frac{2\pi}{3} G \rho_0 R^3 = \left( \frac{\pi}{6} \right)^{1/3} G M^{4/3} r_0^{4/3}.$$

In GR, though the TOV eqn is

$$\frac{dP}{dr} = -\frac{G M_{\text{ch}}}{r^2} \left( 1 + \frac{P}{\rho c^2} \right) \left( 1 + \frac{4\pi r^3 P}{M c^2} \right) \left( 1 - 2GM/rc^2 \right)$$

which integrates to

$$P = \rho c^2 \left[ \text{something complicated} \right]$$

and

$$P_c = \rho c^2 \left[ \frac{1 - (1 - 2GM/rc^2)^{1/2}}{3(1 - 2GM/rc^2)^{1/2} - 1} \right].$$

Then, have $P_c < c^2$ only if

$$\frac{GM}{Rc^2} < \frac{\sqrt{3}}{4},$$

which is a slightly stronger bound than $\frac{M}{Rc^2} < 2$ from the Schwarzschild radius.

For constant density $\rho_0$, this yields

$$M < \frac{8}{27} \left( \frac{\rho_0}{G} \right)^{3/2} \left( \frac{\frac{3}{4\pi \rho_0}}{3/2} \right)^{1/2}$$

$$= 7.5 \left( \frac{\rho_0}{\rho_{\text{crit}}} \right)^{1/2} M_{\odot}$$
A slightly stronger bound to $M/R$ is obtained by consistency causality, which requires $C_5^2 = (\partial P/\partial \rho) \leq C^2$, or $P = \rho c^2$. This results in

$$\frac{GM}{R_c^2} < \frac{2GM}{2G} = 1$$

Pulsar spin-down:

Pulsars are rapidly rotating NSs that are to produce pulsed signals with periods ~ms—secs. E.g., the Crab pulsar in the Crab nebula from a SN in 1054 AD. It has period $P = 33$ ms and is slowing down with

$$\frac{dP}{dt} = \frac{5}{90 \times 10^5}$$

The rapidity of the pulses identifies pulsars as NSs. A star has a max spin freq $f_{\text{max}}$ from

$$\frac{GM}{R_s^2} = 2\pi f_{\text{max}}^2 \Rightarrow P_{\text{min}} = \frac{3\pi}{2\pi} f_{\text{max}} = 2\pi \left(\frac{R_s}{6}\right)^2 \sim 10^3$$

or $P_{\text{min}} = 10^{-4} \left(\frac{R_s}{m/M_0}\right)^{3/2}$ sec

for $M \sim M_0$. $P \sim P_{\text{max}} \Rightarrow R \lesssim 10^{12} R_0 \sim 100 \text{ cm} \sim 60 \text{ km}$

If we approximate NS by uniform density sphere, then the moment of inertia is

$$I = \frac{2}{5} MR^2 = 2.5 \times 10^{45} \left(\frac{M}{1M_0}\right) \left(\frac{R}{1.4\times10^6\text{ cm}}\right)^2 \text{ g-cm}^2$$

The Crab pulsar slows down at a rate

$$\frac{dw}{dt} = -2.4 \times 10^{12} \text{ sec}^2$$

implying a spin-energy loss rate

$$\frac{dE_{\text{el}}}{dt} \sim \frac{dP}{dt} \sim 4.6 \times 10^{38} \text{ erg/sec}$$

which is comparable to the Crab nebula luminosity, $5 \times 10^{38} \text{ erg/sec}$
The central equation for the dipole is magnetic dipole radiation. If it has the B-field, a B dipole oriented from the axis of the neutron star, it radiates with

\[
\frac{dE}{dt} = \frac{2}{3} c^3 m^2 w^4 \sin^2 \theta \quad \left( = \frac{2}{3} c^3 |\mathbf{u}|^2 \right),
\]

where \( m = \frac{B^2 r^3}{8} \) is the magnetic dipole which is required for Crab to be \( m \approx 3 \times 10^{-6} \) cm³.

\( B \approx 4 \times 10^{13} \) G or

\( B \approx 10^{12} \) G.

This seems large, but magnetic fields can be enhanced by large tides if initially magnetized pre-collapse iron core contains the B field as it collapses.

If B-dipole radiation is at work in the Crab, then

\[
\frac{dE}{dt} = 1w dw / dt \times w^4 \quad \text{or} \quad \frac{dw}{dt} = -C w^3
\]

where \( C = 3.5 \times 10^{-16} \) sec⁻¹ for \( w = 1905 \) s⁻¹ \( \frac{dw}{dt} = -2.4 \times 10^{-5} \) s⁻².

Integrating,

\[
t = \frac{1}{2C} \left[ \frac{1}{w^2} - \frac{1}{w_i^2} \right] < \frac{1}{2Cw_i^2} = 1253 \text{ years},
\]

as opposed to historical age \( (\approx 950 \text{ yrs}) \) since SN.

The braking index \( n \) is defined to be

\[
n = \frac{-w_i w}{w^2},
\]

It is \( n=3 \) for magnetic-dipole model and \( n>5 \) for emission of GWs. Measured to be 2.515 for Crab and 2.85 for PSR 1509-58.
Suppose the NS has a dipole aligned with spin axis:
\[ \mathbf{B}_{\text{dip}} = \mathbf{B}_p \mathbf{r}^2 \left( \frac{\sin^2 \theta}{r^3} \hat{e}_r + \frac{\sin \theta \cos \theta}{r^2} \hat{e}_\theta \right). \]

Inside NS, there are enough free e's so the MHD approximation holds, i.e., the e's rearrange themselves so that they are not accelerated— they short out the fields. I.e.,
\[ \mathbf{E}^{(\text{in})} + \frac{\mathbf{c} \times \mathbf{B}}{\mathbf{c} \cdot \mathbf{B}} \times \mathbf{E}^{(\text{in})} = 0. \]

Thus, just inside the surface, \[ \mathbf{E}^{(\text{in})} = \frac{R \mathbf{B}_p \sin \theta}{c} \left( \frac{\sin^2 \theta}{2} \hat{e}_r - \frac{\sin \theta \cos \theta}{c} \hat{e}_\theta \right). \]

The \( \hat{r} \) component of \( \mathbf{E} \) appears outside may jump at the NS surface if charges accumulate there but the tangential component is constant. Thus, outside the star,
\[ \mathbf{E}^{(\text{out})} = -\frac{\partial}{\partial t} \left( \frac{R \mathbf{B}_p \sin \theta}{2c} \right) = \frac{3}{2} \left[ \frac{R \mathbf{B}_p}{3c} \mathbf{P}^{(\text{out})} \right]. \]

Outside the star, \[ \frac{\mathbf{E}^{(\text{out})}}{\mathbf{E}} = -\mathbf{\nabla} \Phi \] with \( \mathbf{\nabla} \mathbf{\nabla} \Phi = 0 \), implying\( \mathbf{\nabla} \mathbf{\nabla} \mathbf{\nabla} \mathbf{\nabla} \mathbf{\nabla} \mathbf{\nabla} = 0 \) to satisfyIBC.

i.e., a quadrupole electric field is induced.

Inside the star, there is a charge density \( \rho_e = \frac{\dot{Q}}{4\pi} \) \( \mathbf{E} = \frac{\dot{Q}}{4\pi c} \mathbf{B} \)
or \( \rho_e = 7 \times 10^4 \mathbf{B}_e \mathbf{P} \mathbf{\text{cm}}^{-3} \).

At the surface of the star, there is a surface charge density \( \sigma = \frac{1}{4\pi} (\mathbf{E}^\text{out} - \mathbf{E}^\text{in}) = -\mathbf{B}_p \frac{2R \cos^2 \theta}{4\pi c}. \)

Inside, \( \mathbf{E} \cdot \mathbf{B} = 0 \), but outside (assuming vacuum outside),
\[ \mathbf{E} \cdot \mathbf{B} = -\frac{R \mathbf{B}_p}{c} \left( \frac{R^2}{r} \right) \mathbf{B}_p \cos^2 \theta. \]

Therefore, etc.
\[ E = \frac{2R_0}{c} - 2\pi J_\ell R_0 V_{\text{lin}}. \]

This is much larger than typical e.g., for protons:

- **Electric** \[ \frac{eR_0 B_0 c}{GMm/R^2} \approx 10^8 \gg 1 \]

and is \( \approx 10^9 \) for e's...

\[ \Rightarrow \text{Vacuum is unstable!} \]

\[ \Rightarrow \text{Particles are stripped from surface of NS, and } \text{NS is necessarily surrounded by plasma of charged particles tied to } B\text{-field lines; magnetosphere.} \]

Resulting picture is that \( B\)-field and plasma co-rotate with NS (like an egg beater) out to light cylinder of radius \( R_c = c/\Omega = 5 \times 10^6 \) pc beyond which co-rotation would imply \( V \gg c \) for plasma. What happens at larger radii is unclear but probably involves bending back of field lines to toroidal component, and perhaps expulsion of highly relativistic particles.

Within light cylinder, \( E \cdot B = 0 \), and \( E = \frac{(c^2 B_0)^2}{c} \).

Thus \( E \parallel B \) with \( |E| \approx |B| \) at light cylinder, thus driving Poynting flux

\[ J \sim E \cdot B \sim \frac{c p^2}{4\pi} \quad \text{over area } \sim 4\pi R_c^2, \text{ or} \]

luminosity

\[ \frac{dE}{dt} \sim B_0^2 R_c^3 c \sim c \left( \frac{B_0 R^3}{R_c^3} \right) R_c^2 \]

\[ \sim \frac{B_0^2 R_c^2 L^2}{c^2} \]

Therefore, get some spin-down from pulsar magnetosphere as in more magnetic-dipole model, even if \( B \text{ still } \text{in SW.} \)
If particle energy $E_p \gg \frac{B^2}{8\pi}$, then $B \propto \frac{1}{r}$.

$B \sim 10^{-4} \text{ G @ } r = \text{pc};$ too small.

The picture of the pulsar magnetosphere is:

Angular size of polar cap is $\Theta_p$, determined by noting that $\sin^2 \Theta / R$ is constant along the dipole field lines. Thus, $\Theta_p \sim \sqrt{R \Omega / c}$, and the polar cap has area

$$A_p \sim \pi R^2 \Theta_p \sim \pi \left( \frac{2R^3}{c} \right)$$

Pulsar emission mechanisms:

In spite of huge amount of detailed data, these are not well understood. In part this is because pulsed radio emission is "dive" --- constitutes $\lesssim 10^{-4}$ (e.g. in Crab) of total emission.
From observation with the Jodrell and Oakley observatories, an $S_{15}$ at the equator was $\lesssim 10^{3}$ cm$^{-2}$.

Implies $T \sim 10^{10} - 10^{13}$ K, $10^{-4} - 10^{-1}$ V.

$\Rightarrow$ not spontaneous emission.

One fourth, universal ingredient:

Particles accelerated by E field will cross become very energetic. E.g. in polar-cap, the voltage is

$$V \sim \frac{R_{O}}{c} B \Omega \sim 10^{6} \text{ V} \quad \text{for} \quad \Omega \sim 10^{14} \text{ Hz}, B \sim 10^{12} \text{ G} \\
\Rightarrow \text{Mev/keV}$$

Thus $e^{-}$s attain $\gamma \gg 1$. They then produce curvature radiation as they cross B field lines. These $\gamma$-rays then escape (high-energy emission in polar-cap models) and/or run into other $\gamma$s (or B-field) to produce $e^{+}e^{-}$ pairs. The magnetosphere is therefore populated by a pair plasma.

$\Rightarrow$ may also be large-scale charge separation; this may then allow for coherent emission (where intensity scales as $N^{2}$ rather than $N$) which may explain $T_{\mu} \gg 10^{8}$ K.

In polar-cap models, all this damage place near poles, and emission is along axis of symmetry.

In outer-gap models (or light-cone models), emission occurs near light cone near equator in NSs with misaligned B fields.
Equation (1):
\[ \tau = \frac{P}{2P} \]

Equation (2):
\[ B_{\text{field}} = \frac{I}{2\pi R} \left( \frac{M}{2R^2} \right) \]

Evolutionary tracks:

Death line: voltage drop \( \propto B \rho^2 \propto P^2 \)

(5) Spin-up limit. Pulsars in binaries do not follow some rules and occupy some regions of \( P \rho \) space. May be spin up by accretion from companion. A neutron star is expected to have

\[ P \approx 2 \left( \frac{B}{10^9 G} \right) \text{ ms} \]
\[ E_{\text{star}} = \frac{GM^2}{R} \approx 5 \times 10^{10} \text{ erg} \approx \frac{1}{5} M_c \text{ eV} \]
\[ g \approx \frac{GM}{R^2} \approx 2 \times 10^6 \text{ cm sec}^{-2} \]
\[ \bar{\rho} \approx \frac{4\pi M}{4\pi R^3} \approx 7 \times 10^7 \text{ g/cm}^3 \approx 2-3 \times \text{normal nuclear density} \]

Structure: atmosphere, outer crust, inner core, outer core, inner core

Atmosphere: \( \sim 10 \text{ cm for } T \sim 3 \times 10^6 \text{ K} \)
- \( \sim \text{mm for } T \sim 3 \times 10^5 \text{ K} \)
- models not complete for \( T \lesssim 10^6 \text{ K} \)
- \( \rho \lesssim 10^6 \text{ g/cm}^3 \); negligible mass; primarily \( ^{56}\text{Fe} \)
- shapes emergent spectrum
- plasma layer: highly ionized; atomic levels distorted by high \( B \) fields

Outer Crust: (outer envelope)
- from atmosphere to \( \rho \approx 4 \times 10^{11} \text{ g/cm}^3 \)
- \( \lesssim 100 \text{ s M thick} \)
- \( \text{ions}^+ \text{e}^- \text{s} \) (magnetic effects may be important)
- top few m's is non-degenerate \( \text{e}^- \text{gas} \)
- deeper \( \rightarrow \text{degenerate \ e}^- \text{gas, ultra-relativistic} \)
- solidifies at high depth
- \( \beta \) capture in nuclei \( \rightarrow \text{n-rich nuclei} \)
- bottom of crust: \( \text{n's start to drop, form n gas} \)
  \( \rho \approx 4 \times 10^{10} \text{ g} \)
- \( \text{n gas} \) is still background sea for nuclei

Inner Core: \( \sim 1 \text{ km thick} \)
- from \( \rho \approx 4 \times 10^{10} \text{ g/cm}^3 \) to \( \rho \approx 0.5 \rho_0 \)
- magnetic effects not important
- \( \text{e}^- \text{s}, \text{n-rich ions, n's} \)
- \( \text{n fraction increases as } \rho \text{ nucleon increases as } \rho \text{ nuclei} \rightarrow 0 \text{ at crust/core boundary} \)
- nucleons can be superfluid if \( T \) low enough
In the core of a white dwarf, where nuclei cease to exist
\[ \frac{\rho}{\rho_{\text{crit}}} \]

n-p ratio in \( \beta \) equilibrium

Many-body effects may be important
everything very degenerate
nucleon-nucleon interactions \( \Rightarrow \) n's superfluid
p's superconducting
contains (with inner core) \( \approx 99\% \) of mass

Inner core:

\[ \rho \approx 2\rho_0 \]
\[ R \approx \text{few km} \]

Composition/EoS uncertain; nobody knows what nuclear matter at such high \( N/N_p \) does
hypers? pions? kaons? quark matter?

may be probed in heavy-ion collisions
(e.g., RHIC @ Brookhaven)

Superfluidity: occurs for \( T < T_c \) = critical temperature (uncertain)
does not affect EoS, M-R much
n's in inner crust should be superfluid
affects heat capacity and \( \gamma \) emission, and so is relevant for cooling of NS.

Spin of NS is quantized in discrete vortices
superfluid p's are superconducting
B fields quantized in flux tubes
B fields fix crust/core rotation to be same
but superfluid neutrons may have different spin.
The stars are born with $T = 10^5 K$ and radiate $\sim 10^{33}$ ergs in a few secs. $T = 10^6 K$ after a day.

$X$ emission dominates for first $10^{-4}$ yr to $10$ K; then $X$ radiation from surface takes over.

Observationally NS surface temperatures are $\sim 5 \times 10^5 K$  

$\Rightarrow$ thermal emission in UV/soft-X-ray

$\gamma$'s produced with $E_{\gamma} \sim k_B T$ and scatter from $N$ with $\sigma \sim 4.4 \times 10^{-45}$ cm$^2$ ($E_{\gamma}$/MeV$^2$),

so their mean-free path is

$$l = \frac{1}{\sigma} \approx 2500 \text{ km} \left( \frac{R}{12 \text{ km}} \right) \left( \frac{1.4 \text{ MeV}}{m} \right) \left( \frac{10^5 K}{T} \right)^3$$

$\Rightarrow$ at $T=10^6 K$ is opaque, but transparent for $T=10^5 K$  

Direct URCA process:  $n \rightarrow p + e^+ + \nu_e$; $p + e^- \rightarrow n + \nu_e$  

is slow if $p$'s are scarce; slowed also by Pauli blocking  

Modified URCA process:

$\gamma + (n, p) \rightarrow p + (n, p) + e^- + \bar{\nu}_e$  

$\bar{\nu}_e + (n, p) \rightarrow n + (n, p) + e^+ + \nu_e$  

can increase rate by providing more energy-momentum combinations.

Measurements of $t$ (e.g. from $P/P$) and $T_{eff}$ (from X-ray) suggest modified URCA is important.
Some pulsars occasionally suddenly speed up and then slow down. This is explained by starquakes, cracks in the crystalline crust. Moment of inertia \( I \)\( \downarrow \) sending \( \Delta I \). E.g., \( \Delta \Omega / \Omega \lesssim 10^{-6} \) once in awhile. More characteristically, \( \Delta \Omega / \Omega \lesssim 10^{-6} \).

After glitch, pulsar spins down more rapidly. This is explained by the time it takes crust to spin superfluid-H component up. Then asymptotes to usual B-field spindown with slightly smaller \( I \) that results from quake.

Magnetars:

Typical NS B fields are \( 10^{12} \) G. But a few have \( 10^{14} - 10^{15} \) G \( \Rightarrow \) magnetars.

If \( B \) is large enough, then the Compton radius \( \lambda = \frac{\hbar}{mc} \)
of an \( e^- \) becomes smaller than the de Broglie wavelength, \( \lambda = \frac{\hbar}{p} \). Thus, above a quantum-critical field

\[
B_{QC} = \frac{M_e c^3}{\hbar e} = 4.4 \times 10^{13} \text{G,}
\]

QM effects become important.

Example: Soft Gamma Repeaters (SGRs).

are x-ray sources that repeatedly emit x-ray flashes; spectrum is softer than GRBs.

Have \( P \approx 5\text{s} \quad P \approx 7 \times 10^{-11} \)
SNR has age $t \approx 10^6$ yr and $P \approx 8 S$

Setting $\frac{P}{2\pi} = t \Rightarrow P \approx 10^{11} \Rightarrow B \approx 10^{-6} G$

Giant flares:
1806-20

SGR 0526-66's most energetic burst was 27 Dec '04
with $E \approx 2 \times 10^{44} \text{erg}$; (typical bursts are $\approx 10^{42} \text{erg/sec}$)
most released in $\approx 1 \text{sec}$ burst with short rise time
rest released in softer pulsating tail

From PP, $B \approx 1.6 \times 10^{15} G.$

$E_{\text{mag}} \sim \frac{B^2}{8\pi} \frac{4}{3} \pi R^3 \sim 10^{45} \text{ erg}$

so burst is only small fraction of $B$-field reservoir.

Explanation for pulsed emission is that starquake re-arranges $B$-field produced X-e+ fireball confined to magnetosphere.

**Anomalous X-ray Pulsars:** (AXPs)

Isolated pulsars emitting pulsed X-ray with $L \approx 10^{35-36} \frac{\text{erg}}{\text{sec}}$
and $P \approx 5-12 \text{ sec},$
$L$ is too high for magnetic dipole; is assumed/guessed to be powered by dissipation of $B$ field. Uncertain?