The PN expand to become a white dwarf until the core decreases to \(10^{-4} - 10^{-3}\) solar mass. The PN expand at a rate of \(10^6\) cm/s, and thus is dispersed after \(10^3 - 10^5\) yr.

Further evolution of massive stars (\(M > 10^4\) Mo): Subsequent stages of burning proceed (in general), non-exponentially, degeneracy being achieved only when an iron core is reached. There is much mass loss (which is not well understood) so that MS evolutionary paths often converge to those of a \(M \approx 30\) Mo stars. Wolf-Rayet stars are \(M \approx 30\) Mo superluminous stars with high-velocity envelopes and current masses that may be only \(\sim 5\) to \(10\) Mo, associated with \(10^4 - 10^5\) km/sec outflows and bright \(N\)-rich nebulae. The luminosity of post-C burning stars remains close to Ld, and evolutionary tracks in the HR diagram cross-cuts horizontally as subsequent burning stages are ignited and exhausted.

Core collapse: After \(5\) - \(6\) Fe ceases in the core, the Fe core contracts until it is held up by degeneracy pressure. However, additional \(Si\) burning in a shell dumps more Fe onto the core and when the Fe core mass exceeds \(1.4\) Mo, it undergoes gravitational contraction. However, unlike earlier stages of stellar evolution, the reason these Fe-group elements are the most tightly bound, there are no further exothermic reactions to increase the internal kinetic energy to supply increased pressure. Instead, \(8 + ^{56}\)Fe \(\rightarrow\) \(13\) ^{56}\)He + \(4\) n which absorbs \(124.4\) MeV of energy and at higher \(T\), \(e^- + p \rightarrow n + \gamma\) (which escapes), further redumps the pressure available to counteract gravity.
The core keeps collapsing almost freely under gravity in a time $t \approx (3.7 \times 10^{10} \text{ sec}) \times 10^{-3} \text{ sec}$, as soon as the Fe core has reached a density $\approx 10^{12} \text{ g/cm}^3$ and a temperature at $T \approx 10^{8} \text{ K}$, as determined before.

At very slightly higher $T$, $^8\text{Be} \rightarrow 2\text{He} + 2p + 3\text{in},$ releasing $\approx 2.5 \text{ MeV}.$

A $1.4 \text{ MeV}$ core when dissociated absorbs $4 \times 10^{51} \text{ erg}$ from iron photo-disintegration and $10^{52} \text{ erg}$ by the disintegration, for a total of $1.4 \times 10^{52} \text{ erg}$ comparable to the integrated energy output of the Sun over 10 billion years; $2 \times 10^{33} \times 3 \times 10^9 \times 10^9 \approx 10^{52}$.

At (n, p) capture and neutronization:

At sufficiently high $\rho$, $^4\text{He}$ Fermi (total) energy will be $E_F = 1.3 \text{ MeV} = N_\text{F} - N_\text{He}$, and $e^+ + p \rightarrow n + \nu_e$ will begin to convert protons to neutrons. In the iron core, there are no free neutrons, but betatronics occur $e^+ + ^56\text{Fe} \rightarrow ^56\text{Mn} + \nu_e$ (inverse-beta decay) occurs for $E_F \approx 4.2 \text{ MeV}$ giving total energy $KE = 3.7 \text{ MeV}$, which occurs at density $\rho \approx 1.1 \times 10^{12} \text{ g/cm}^3$.

At $\rho = 1.5 \times 10^{12} \text{ g/cm}^3, e^+ + ^56\text{Mn} \rightarrow ^56\text{Ti} + \nu_e.$

At $\rho \approx 10^{12} \text{ g/cm}^3, n \rightarrow p$ becomes very rapid and the "neutronization neutrinos" carry energy away, thereby accelerating the collapse. The typical $\nu$ energy will be

$$E_\nu \approx E_F \left( \frac{\text{MeV}}{\text{mc}^2} \right) = \frac{p}{\text{mc}} \approx \sqrt{\frac{3}{8 \pi \rho}} \left( \frac{p}{\text{MeV}} \right) \approx 10^{-3} \left( \frac{p}{\text{MeV}} \right)^{1/3}$$

or $E_\nu \sim 10 \text{ MeV}$ for $\rho \approx 2 \times 10^{12} \text{ g/cm}^3$. Since there are $\approx 10^{57}$ nnucleons roughly,

$$E_\nu \sim 10^{57} (10 \times 1.6 \times 10^{-13}) = 16 \times 10^{52} \text{ erg}$$

are carried away by neutronization $\nu$'s. As we will see, this is only a small fraction of the total $\nu$ flux. Although the collapse timescale is $\sim 10^{-3} \text{ sec}$, the $\nu$ burst from SN 1987A lasted $\sim 10 \text{ sec}$. This is because at sufficiently high densities, the collapsing core becomes opaque to $\nu$'s.
These $\nu$'s interact primarily by a coherent scattering from
nuclei of mass number $A$ with cross section

$$\sigma_{\nu} = 10^{-45} \text{ cm}^2 \left( \frac{E_{\nu}}{m_{\nu} c^2} \right)^2$$

for $\nu + (2A) \rightarrow \nu + (2A)$

$$= A^2 \left( \frac{E_{\nu}}{m_{\nu} c^2} \right)^2 \times 10^{-44} \text{ cm}^2$$

and since $n = \rho/(4\pi m_{\nu} v^2)$, the mean-free path is

$$\mu \sim \frac{1}{n\sigma_v} = \frac{1}{4\pi \rho (E_{\nu}/c^2)} \times 1.7 \times 10^{-25} \text{ cm}.$$

With $E_{\nu} \approx 2$, $A = 100$, $\rho = 10^7 \text{ cm}^{-3}$ (the core radius)

for $(\rho M_{\odot}) \approx 3.6 \times 10^{-9} \text{ g/cm}^3$. "Detailed calculations" show that $\nu$'s are trapped in the collapsing core for

$p > 3 \times 10^{-26} \text{ g/cm}^3$.

When the core has reached nuclear densities,

$$p = \rho v^2 \approx \frac{34 M_{\odot}}{4 \pi R_0^3} = 2.3 \times 10^{-28} \text{ g/cm}^3$$

(using $R_0 = 1.2 \times 10^6 \text{ cm}$)

The neutrons become degenerate and their pressure halts
further collapse, and a protoneutron star of radius

$R \approx 2 \times 10^6 \text{ cm}$ and mass $M = 1.4 M_{\odot}$ remains. The

gravitational energy released in this collapse is

$$AE_{\text{grav}} = +GM^2/R \approx 3 \times 10^{53} \text{ erg}$$

which is $\approx 10^5$ the amount absorbed by photodisintegration
and neutronization.

When this collapse occurs, the "bounce" drives a
shock into the envelope that expands the envelope. Although the details are not well understood (and probably
require neutron energy transport), observations show that $\approx 10 M_\odot$ of envelope are ejected at velocities

$v \approx 10^4 \text{ cm/sec}$, implying a kinetic energy

$$E_{\text{kin}} = \frac{1}{2} 10 M_\odot v^2 \approx 10^{52} \text{ erg} \ll AE_{\text{grav}}$$

(Note that this is $\approx 10^5$ the envelope binding energy

$$E_{\text{bind}} \approx \frac{GM_{\odot}(M - M_{\odot})}{R} \approx 5 \times 10^{51} \text{ erg}$$.
It is also the gravitational binding energy is also much larger than the optical energy of the SN:

\[ E_{\text{bnd}} \approx L_{\text{SN}} \approx (10^{44} \text{ erg/sec})(3 \times 10^{55} \text{ sec}) \approx 3 \times 10^{55} \text{ erg} \]

Most (>90%) of the binding energy is liberated as \( \nu \)'s.

Neutrinos core into thermal equilibrium in the proto-NS (\( \nu_e, \bar{\nu}_e, \nu_x, \bar{\nu}_x \)) and are emitted from a "neutrinosphere".

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**Diagram:**
- Inner core
- Outer core
- Si burning shell
- \( \nu \) trapping surface
- \( \nu \) photosphere

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We can get a rough understanding of the size of the neutrinosphere by assuming all \( 6 \nu \) degrees of freedom are emitted thermally from radius \( R \). The luminosity

\[ L_\nu = \frac{3 \times 10^{53} \text{ erg}}{10 \text{ sec}} = 3 \times 10^{52} \text{ erg/sec} \]

must then be

\[ L_\nu = \frac{4\pi R^2}{(4/3)\pi} \frac{3}{5} T_\nu^4 \]

\[ = \frac{3}{5} \times 10^{53} \left( \frac{R}{2 \times 10^6 \text{ cm}} \right)^2 \left( \frac{T}{10^6 \text{ K}} \right)^4 \text{ erg/sec} \]

For \( L_\nu = 3 \times 10^{52} \) we get reasonable agreement with what we expect for the radius and \( T \) (note that the detected \( \nu \)'s had energies \( \sim 10 \text{ MeV} \sim 10^9 \text{ K} \)).

Real-world complications include the fact that \( \nu_e \) and \( \nu_x \) have different interaction strengths and so are emitted at higher \( T_\nu \) from smaller \( R \); also the spectrum may not be precisely thermal and the time evolution changes \( T_\nu \), etc on a \( \sim 10^{-6} \text{ sec} \) timescale.
As the shock generated by core collapse propagates through the mantle, it heats the material to $T > 5 \times 10^7$ K for layers below Ne-O layers. At these $T$, NSE is achieved in a few seconds (the dynamical time) producing primarily $^{56}\text{Ni}$. The ejected $^{56}\text{Ni}$ then decays:

\[ ^{56}\text{Ni} \xrightarrow{\beta^{-}} ^{56}\text{Co} \xrightarrow{\beta^{-}} ^{56}\text{Fe} \]

$^{56}\text{Ni}$

\[ 3.0 \times 10^{16} \text{erg/g} \]

\[ \tau_{\beta} = 267 \text{ days} \]

$^{56}\text{Co}$

\[ 6.4 \times 10^{16} \text{erg/g} \]

\[ \tau_{\beta} = 77.1 \text{ days} \]

This powers the light output of the SN during the decaying part of its light curve.

For SN 1987A, $\approx 0.075 M_\odot$ of $^{56}\text{Ni}$ was ejected.

For Type-I SN, where a degenerate CO is blown up, almost all $1.4 M_\odot$ are converted to $^{56}\text{Ni}$.

EGRET on CGRO detected 1.8-MeV $\gamma$'s from decay of an excited state of $^{26}\text{Al}$ with $\tau_{\beta} = 7.2 \times 10^{5}$ yr throughout the galaxy. This provides evidence for ongoing stellar nucleosynthesis from SN. The decay products of $^{26}\text{Al}$ have been detected in solar-system meteorites indicating the solar system was formed from SN products.

Finally, the gamma-ray burst of 29 March 2003 (and to some extent, 1998bw before it) demonstrates that at least some, if not all/most GRBs are SN. The suggestion is that in some SN with rotating progenitors, the core may collapse to a BH rather than a NS, and an accretion disk or torus may form driving jets through the star and producing the GRB along this jet.