Quantum Mechanics (171.605),
Fall 2016

Midterm Exam (Problem Set 7)

Due: 8 November 2016

Rules: The midterm will be like any other homework—you can spend as much
time as you like, and you can refer to the book and to notes. Please do not,
however, work with other students, nor consult with others. Please try to avoid
other resources (online, etc.), but if you do look something up, just be sure to
note that in your solution.

1. Consider a charge $e$ at the origin and a magnetic monopole of magnetic
charge $m$ a distance $R$ away. Calculate the angular momentum contained
in the electromagnetic field. Derive the quantization condition on the
magnetic charge $m$ that arises if the angular momentum is quantized in
units of $\hbar$. Does this agree with the Dirac quantization condition? (You
can refer to an E&M book for this problem and do not need to note that
on your solution.)

2. Consider a particle of mass $m$ constrained to move on a sphere of radius $R$.
Calculate the energy eigenvalues and the associated eigenfunctions.

3. Consider a particle of mass $m$ constrained to move on a sphere of radius $R$
in 4 spatial dimensions (rather than the three spatial dimensions you
considered in the last problem). Calculate the energy eigenvalues and the
associated eigenfunctions.

4. 3.17 in Sakurai-Napolitano

5. 3.20 in Sakurai-Napolitano

6. Extra credit: (Note that this problem does not, as far as I know, have
a solution, or at least not an easy one. There are interesting conceptual
issues that you may be entertained to think about.) Suppose the particle
from problem 2 has an electric charge $e$ and that there is a magnetic field
$B$ in the $\hat{z}$ direction. What are the energy eigenvalues and eigenstates? Is
there anything interesting to say about quantization of magnetic flux?