Particle Astrophysics (171.697), Spring 2017

Problem Set 10

Due: In class, first class of week 11

1. Show that the polarization of radiation scattered from an electron cloud of optical depth $\tau \ll 1$ into the direction $\hat{z}$ is

$$Q - iU = \sqrt{\frac{3}{40\pi}} \tau a_{22}, \tag{1}$$

where a_{22} is the radiation quadrupole moment incident on the electron cloud.

2. Consider a single Fourier mode of of (comoving) wavevector $\vec{k} = k\hat{z}$ in the $\hat{z}$ direction and of (comoving) wavenumber k. It appears in the metric as

$$ds^2 = a^2(\tau) \left[d\tau^2 - dx^2(1 + h_+) + dy^2(1 - h_+) + dz^2 \right]. \tag{2}$$

Show that this equation satisfies (in the absence of anisotropic stress),

$$\ddot{h}_+ + 2aH\dot{h}_+ + k^2 h_+ = 0. \tag{3}$$

Write the solutions during matter domination and during radiation domination. Match the solutions and their first derivatives at matter-radiation equality to write the late-time (well into matter domination) solution for small-wavelength (i.e., those that enter during radiation-domination) modes as

$$h_+(\tau) = h_k(\tau) \sin\left[k\tau + \delta(k)\right], \tag{4}$$

where $h(\tau)$ is slowly varying with τ, and $\delta(k)$ is a phase. Find $h(\tau)$ and $\delta(k)$.

3. Write the tensor amplitude in the previous problem as $h_+(\vec{x}, \tau) = h(\tau)e^{ik\tau}e^{-ikz}$. Show from the geodesic equation that the frequency ν of a photon that propagates through this spacetime is

$$\frac{1}{\nu}\frac{d\nu}{d\tau} = -\frac{1}{2}(1 - \mu^2)\cos 2\phi \, e^{-ikz} \frac{d}{d\tau}\left(he^{ik\tau}\right), \tag{5}$$

where μ is the cosine of the angle between $\hat{z}$ and the photon's direction of motion, and ϕ the azimuthal angle of the photon's direction of motion.

4. Run CLASS or CAMB for a scale-invariant spectrum of primordial gravitational waves (tensor perturbations) and then plot the BB polarization power spectrum. Take a reionization optical depth $\tau \simeq 0.1$. You should see a reionization bump at low ℓ; try to understand the value of ℓ at which this peaks. Try also to understand the value of ℓ at which the recombination peak occurs. You should then see the power spectrum drop with ℓ at high ℓ. Try to derive the power law with which C_ℓ^{BB} falls in this regime. You should also see small oscillations in the high-ℓ power spectrum. Try to understand how those arise (qualitatively, at least, if not quantitatively).