Custom Problem #3:

Show that if $E_x(z,t)$ is the standing wave $E_x = A \cos(\omega t) \cos(kz)$, then $B_y(z,t)$ is the standing wave $A \sin(\omega t) \sin(kz)$.

Custom Problem #4:

Assume a standing wave of the form given in Custom Problem #3. Find the electric and magnetic energy densities and the Poynting vector as functions of space and time. Consider a region of length $\lambda/4$ extending from a node in E_x to an antinode in E_x. Sketch a plot of E_x and B_y versus z over that region at times $t = 0$, $T/8$, and $T/4$, where T is the period. Sketch a plot of the electric energy density, the magnetic energy density, and the total energy density over that region for the same times. Give the direction and magnitude of the Poynting vector S_z for those times.

Custom Problem #5:

Linearly polarized light with polarization direction at angle θ from $\hat{x}$ is incident on a piece of polaroid with easy axis along $\hat{x}$. The first polaroid is followed by a second polaroid with its easy axis along the direction of polarization of the original incident light. Show that if the input intensity is I_0 (intensity means energy flux per unit area per unit time), then the output intensity is $I_0 \cos^4\theta$.

Custom Problem #6:

A very large number $N+1$ of polaroids are arranged as a sandwich. The angle of the easy axis of each polaroid is a constant angle θ greater than that of its immediate predecessor in the sandwich. Thus the last polaroid is at an angle $\square =$ $N\square$ from the first. Neglecting any losses due to reflection at the many surfaces, and supposing that linearly polarized light of intensity I_0 is incident on the first polaroid with its polarization direction along the easy axis of the first polaroid, find the output intensity. Take N to be very large and retain only the first interesting terms in an appropriate power series (Taylor's expansion).

ANSWER: $I = I_0 \square 1 \square \frac{\square^2}{N} + \ldots \square$

This means that even if $\square$ is 90 degrees so that the first and last polaroids are crossed, the output intensity is equal to the input intensity if we have enough intermediate polaroids. Thus, we can "gently" rotate the plane of polarization and lose nothing! This kind of "optical activity" is found in many organic molecules such as sugar, which rotates the plane of polarization without absorbing energy.