

# Physics 171.201 Midterm Exam 1

October 8<sup>th</sup>, 2003

Answer all **four** problems. Be sure that you pace yourself so that you have enough time to work on each problem. Note that the problems do not have equal weight. Partial credit will be given, so be sure to **show your work** as clearly as possible. Good luck!

### List of potentially useful formulae

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (v/c^2)x}{\sqrt{1 - v^2/c^2}}$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y' = \frac{u_y \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$E = mc^2$$

$$\vec{p} = m\vec{v}$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$c^2 t'^2 - x'^2 - y'^2 - z'^2 = c^2 t^2 - x^2 - y^2 - z^2$$

$$E^2 - p^2 c^2 = E'^2 - p'^2 c^2$$

1. (30 points) A rod of proper length  $L_0$  is at rest in a reference frame  $S'$ . It lies in the  $(x',y')$  plane and makes an angle of  $\sin^{-1}\left[\frac{3}{5}\right]$  with the  $x'$  axis. If  $S'$  moves with constant velocity  $v$  parallel to the  $x$  axis of another frame  $S$ :

(a) What is the value of  $v$  if, as measured in  $S$ , the rod is at  $45^\circ$  to the  $x$  axis?

(b) What is the length of the rod as measured in  $S$  under these conditions?

2. (15 points) An event  $A$  occurs in a reference frame  $S$  at the origin in space and time,  $x = 0$  and  $t = 0$ . A second event  $B$  occurs in this reference frame at  $x = 4$  meters and  $t = 2 \times 10^{-8}$  seconds. Is there a reference frame  $S'$  in which these events occur at the same place? Support your answer with a Minkowski diagram. Recall that the speed of light is  $c = 3 \times 10^8$  meters/second.

3. (15 points) Two reference frames  $S$  and  $S'$  move with speed  $c/2$  with respect to each other. Clocks in the two frames are adjusted so that  $t = t' = 0$  at  $x = x' = 0$ .

(a) Draw the Minkowski diagram relating these two systems. Let the axes of  $x$  and  $ct$  for  $S$  be at right angles in your drawing.

(b) Plot roughly the positions of the following points in spacetime:

(A)  $x = 1, ct = 1$ ; (B)  $x' = 1, ct' = 1$ ; (C)  $x = 2, ct = 0$ ; (D)  $x' = 2, ct' = 0$ .

4. (40 points) Consider a new rocket technology in which the rocket is propelled to large velocities by emitting light (ie. photons) as fuel. Recall that photons are particles with zero rest mass. A rocket starting at rest on earth with a rest mass of  $M_0$ , uses this technology to reach a velocity  $0.5c$ .

(a) Determine the final rest mass of the rocket, in terms of  $M_0$ , assuming that all of the lost mass has been converted into fuel.

(b) Determine the total energy of the emitted photons.

