Lecture Notes on Relativity

- Last updated 10/1/02
- Pages 1 – 65
- Lectures 1 – 10

Special Relativity: Introduction

- Describes physics of fast motion
  - i.e. when objects move relative to each other at very high speeds, (v close to c)
  - Note that $c = 3 \times 10^{8}$ m s^{-1} is a huge velocity
    - Compare with space shuttle, for which v ~ 7000 m s^{-1}
      ➔ $v/c = 2.5 \times 10^{-5}$ ➔ classical physics OK
  - But as v → c, classical (Newtonian) mechanics doesn't work. New description is needed: special theory of relativity worked out by Einstein in 1905
    - Notions of space, time and energy profoundly altered
    - Yielding counterintuitive and perplexing predictions that have been verified experimentally

Fundamental concepts: events

- We start by dealing with kinematics, the description of motion
- Key concept: an event is something with a location in space and time
  - location can be written (x,y,z,t)
  - example: baseball struck by bat
- Motion of object can be described by a series of events
- To observe events, we need to define a “reference frame”

Inertial Reference Frames

- A free-floating observer (who is not subject to any ***net force***) defines an inertial reference frame
- Postulates of special relativity
  1. Laws of Physics the same in all inertial reference frames ("Principle of Relativity"):
     - There is no "preferred" IRF
  2. Speed of light is the same for all observers (independent of velocity of source or observer)
     - Speed of light follows from Maxwell's Equations

Constancy of speed of light

- Weird in context of classical physics
  - Suppose a baseball pitcher is travelling in an open-top sedan at 50 mph. He pitches a 90 mph fastball in the forward direction: speed relative to ground will be (almost exactly) 140 mph.
  - But if he sends a light pulse forward from a flashlight at c, the speed relative to the ground will still be c.

L02 →

- This constancy follows directly from Maxwell’s Equations
  - They yield an expression for speed of light that is independent of the source’s motion
  - Unlike sound waves, there is no medium in which the wave propagates (i.e. no “ether”)

The notion of invariance

- Start with a “classical” analogy: the parable of the two surveyors
  - Two surveyors use different reference systems (one using magnetic North, one using geographic North)
  - Coordinates of particular points are different: $(\Delta x, \Delta y)$ and $(\Delta x', \Delta y')$
  - But distance between any pair of points is INVARIANT: $d = (\Delta x^2 + \Delta y^2)^{1/2} = (\Delta x'^2 + \Delta y'^2)^{1/2}$
  - or, in 3-dimensions: $d = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$

The Invariant Interval

- Turns out that for a pair of events, the distance $D = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$ is NOT the same for observers in any IRF
- Furthermore, Δt is not invariant!
- The invariant quantity is the **interval**: τ
  - defined by $\tau^2 = c^2 \Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$
  - invariance of τ implies that light beam has the same speed in any IRF

Lightlike intervals

- The interval between two events is said to be **lightlike** if $\tau^2 = 0$.
  - Example:
    - Event A = emission of laser pulse
    - Event B = pulse received at distant detector
    - In any IRF, $\Delta t = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} / c$ (because c is the same in any IRF)

      $\Rightarrow \tau^2 = c^2 \Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = 0$
  - Invariance of speed of light (2nd postulate of SR) is guaranteed by the invariance of the interval

Three types of interval

- Lightlike
  - $\tau^2 = 0$
  - There is no IRF in which the events appear either simultaneous or in the same place
- Spacelike
  - $\tau^2 < 0$
  - There is an IRF in which the events appear simultaneous
- Timelike
  - $\tau^2 > 0$
  - There is an IRF in which the events appear to occur in the same place

Analogy to distance

- Interval looks a bit like a 4-dimensional analog of distance: talk about the geometry of spacetime
  - but note the minus sign in front of the spatial coordinates
  - and note the appearance of c in front of the temporal coordinate
    - c is fundamental constant that would be critical to kinematics even if light did not exist

L03 →

Measuring the coordinates of an event

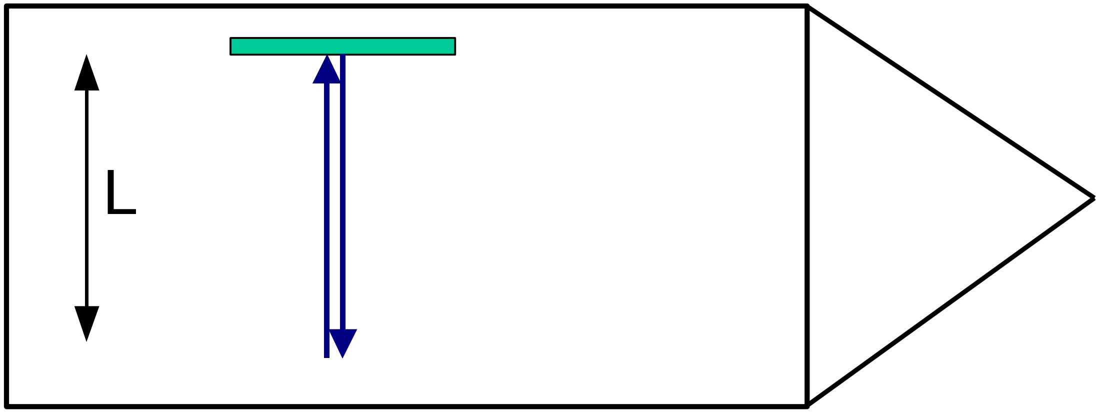
- Key principle: define quantities **operationally** – by saying how you would measure them
- Introduce an (imaginary) lattice of clocks and use it to measure x,y,z,t
    - (see TW, Figures 2-6 and 2-7)

Synchronize clocks using light pulses

- Send out pulse from (x,y,z) = (0,0,0) at time t=0
- Clock at location (x,y,z) is pre-set to $t=(x^2+y^2+z^2)^{1/2} / c$ and started once the light pulse is received
- Each IRF has it own (unique) set of clocks
- For an event to be observed, its coordinates must be reconstructed afterwards from clock records

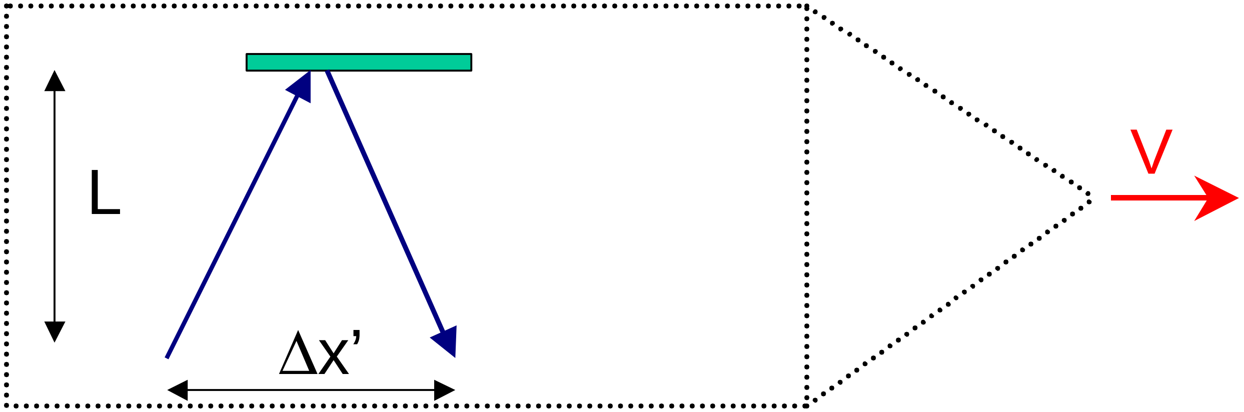
Proof that the interval is invariant (1-D)

- Use a thought experiment
  - Inside a rocket, light pulse leaves source (Event A), bounces off mirror and hits the source (Event B).
  - In this frame, $\Delta t = 2L/c$ and $\Delta x=0$, so the interval is given by $\tau^2 = (2L)^2$ (timelike)



Interval is invariant (contd)

- As viewed from the ground, the light path is longer ➔ $\Delta t' > \Delta t$. Also, $\Delta x' > \Delta x = 0$



- In this frame, $c\Delta t' = 2(L^2 + [\Delta x'/2]^2)^{1/2}$

  $\tau^2 = (2L)^2$ as before

Time dilation

- See quickly that $\Delta t' > \Delta t$ (because light has further to travel in frame of ground and its speed is constant)

$$c\Delta t' = 2(L^2 + [\Delta x'/2]^2)^{1/2}$$

$$\Rightarrow \; c^2(\Delta t')^2 = 4\,[\, c^2(\Delta t/2)^2 + (v\Delta t'/2)^2 \,]$$

$$\Rightarrow \; (\Delta t')^2(1 - v^2/c^2) = (\Delta t)^2$$

$$\Rightarrow \; \Delta t' = (1 - v^2/c^2)^{-1/2}\,\Delta t$$

The Lorentz factor $\gamma = (1- v^2/c^2)^{-1/2}$

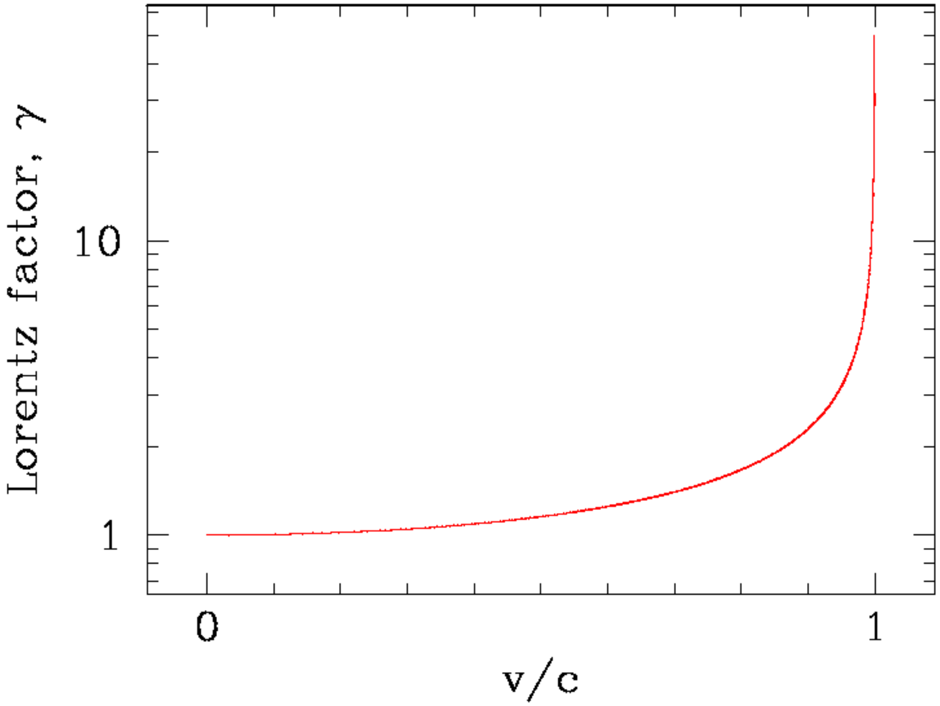
- The moving clock runs slower by a factor $\Delta t'/\Delta t = (1- v^2/c^2)^{-1/2} = \gamma$
  - If $v/c \ll 1$, then $\gamma \sim 1$ ➔ effect negligible
  - As $v \rightarrow c$, γ tends to infinity
- General result: time between events is shortest in the IRF in which their spatial separation is zero.
  - This is called the proper time

Behavior of Lorentz factor

- Limit of small v/c

$$\gamma = (1-v^2/c^2)^{-1/2} = 1 + ½\, v^2/c^2$$

- Figure:



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L04 →

Moving clocks run slow

- Suppose I am in the rocket, wearing a wristwatch with a second hand that ticks (advances) exactly every second.
  - You measure the time between two pulses as 1.0001 s. You say my watch runs slow.
  - You are wearing an identical wristwatch. I measure the time between two pulses of your watch as 1.0001 s. I say your watch runs slow.
  - Who is right? Both of us!

The Relativity of Simultaneity

- Suppose you and I synchronize our watches as I pass you in the rocket
  - Call this Event A
  - The next tick of my watch (1 sec later, according to me) is Event B
  - The next tick of your watch is Event C
- I say that Event B occurs before C
- You say that Event C occurs before B
- **Order of events can depend upon IRF**

Experimental evidence for time dilation

- Decay of cosmic ray muons
  - – Created in upper atmosphere by cosmic ray collisions
  - – Travel downwards at v close to c
  - – Half-life for decay = 1.5 μs
  - – Should travel median distance of only 450 m, but reach Earth's surface
  - – Decay time increased by factor $\gamma \sim 100$
  - – In muon frame, distance travelled is indeed only 450 m: "Lorentz" contraction

Relativistic transformations

- If an event has coordinates (x',y',z',t') in inertial reference frame S' ("the rocket frame") moving at velocity $\underline{v}$ with respect to the "lab frame" S, what are its coordinates in frame S
- Newtonian physics answer: If S' is moving in the positive x direction

$$t = t'$$

$$x = x' + vt$$

$$y = y'$$

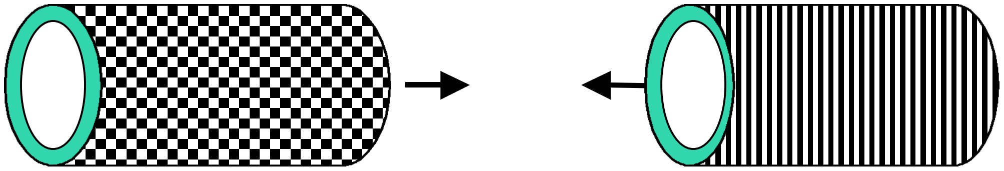
$$z = z'$$

("Galilean transformations")

Invariance of transverse dimensions

- As we've already seen, the Galilean transformations do not keep the interval invariant, or the speed of light constant.
  - ➔Transformations for x and t must be modified

    However, the "transformations" for y and z do hold
    - If y y' or z z' inconsistencies would arise, e.g. in analysing the case of two colliding pipes (assumed implicitly in our proof of the invariance of the interval)



Lorentz transformation (1)

- To preserve the invariance of the interval, we must modify the transformation for x and t.

$$t = Bx' + Dt'$$

$$x = Gx' + Ht'$$

  - This is the most general linear transformation
  - Why **linear**? Because otherwise the results would depend on the arbitrary choice of origin.

Lorentz transformation (2)

- Consider an event at time t and location $x'=0$ in frame S' (the rocket frame).
- In S, location $x = vt$.
- Invariance of interval

$\tau^2 = c^2t^2 - x^2 \quad = c^2t^2 - v^2t^2$ (frame S)

$\tau^2 = c^2t'^2 - x'^2 = \quad c^2t'^2$ (Frame S')

➔ $t^2 (1-v^2/c^2) = t'^2$

➔ $t = \gamma\, t'$ ➔ value of D is γ

Also $x = vt = v\, \gamma\, t'$ ➔ value of H = $v\, \gamma$

Lorentz transformation (3)

- So far, we have

$$t = Bx' + \gamma t'$$

$$x = Gx' + \gamma v t'$$

- Now insist that the interval is invariant for **any** event
  - Find $B = v G / c^2$ and $G^2 - c^2 B^2 = 1$

→ $G = \gamma$

$B = v \gamma / c^2$

Lorentz transformation (4)

- Final result:

$$t = \gamma\,(vx'/c^2 + t')$$

$$x = \gamma\,(x' + vt')$$

$$y = y'$$

$$z = z'$$

L05 →

Inverse Lorentz transformation

- Solve for x’ and t’ in terms of x and t

$$t' = \gamma\,(-\,vx/c^2 + t)$$

$$x' = \gamma\,(x - vt\,)$$

$$y' = y$$

$$z' = z$$

- As expected, this is the same as the transformation from S’ → S but with the sign of v changed

Addition of velocities

- Suppose an object starts at the origin at t'=0 in the rocket frame and travels at speed u in the +x' direction.
  - At time, t', it is at location x'=ut'
  - Applying the Lorentz transform

$$(x,t) = (\gamma[ut' + vt'], \gamma[vut'/c^2 + t'])$$

➔ In S frame, velocity is x/t

$$= \frac{(u + v)}{(1 + uv/c^2)}$$

Lorentz contraction

- A rocket travelling is travelling at velocity v in the x-direction? If its length in the rocket frame (“proper length”) is L_0, what is the length measured in the lab frame
  - Event A = location of front of rocket at time t
  - Event B = location of back of rocket at same time t
- Length measured in lab frame = Δx
- Length measured in rocket frame,

$$\Delta x' = \gamma\,(\Delta x - v\Delta t) \;=\gamma\,\Delta x = L_0 \quad \text{(inverse L.T.)}$$

- Hence, $\Delta x = L_0 / \gamma$

Pole in the barn paradox

- Doesn't this appear to violate the symmetry between two inertial frames?
  - We argued previously that transverse dimensions couldn't change because we would obtain inconsistent results in the case of the colliding pipes
  - Consider the following: a pole vaulter with a 20 m pole runs at $v = (3/4)^{1/2}$ c (i.e. with Lorentz factor γ=2) through a barn of length 10 m.
    - Observer in lab frame says he can close the barn doors simultaneously for an instant while the pole is inside, because pole's length contracts to 10 m so it just fits
    - But pole vaulter says the **BARN** is shortened to 5m: she won't fit!

Pole in the barn:solution

- Word used carelessly: “simultaneously”
  - Closing of doors is simultaneous in lab frame (S) but not in the pole vaulter’s frame (S’).
    - Front door closed at t = t’ = 0
    - Back door closed at t = 0 ➔

      $t' = \gamma(-v\, x/c^2 + t\,) = \; -\, 2.\ (3/4)^{1/2}\ (-\ 10\text{m})\ /c$

      (**AFTER** closure of the front door)
    - Allows pole to travel an additional distance $vt' = 2\ (3/4)\ (10\text{m}) = 15\text{m}$ ➔ just fits!

L06 →

The twin paradox

- Time dilation would appear to violate the symmetry between two IRF.
  - Two twins part company in 2002
    - Sister travels to Canopus (99 light years away) at speed v/c = 99/101 and then return to Earth at the same speed (➔ $\gamma = 5.05$)
    - In Earth frame trip takes 101 yr each way
      - Sister returns in Year 2204
      - Twin brother is long dead
    - In rocket frame, proper time is 101 yr/ 5.05 = 20 yr each way
      - Sister has only aged 40 years

Twins are not symmetric

- From the sister's point of view, hasn't the brother travelled away and therefore aged more slowly?

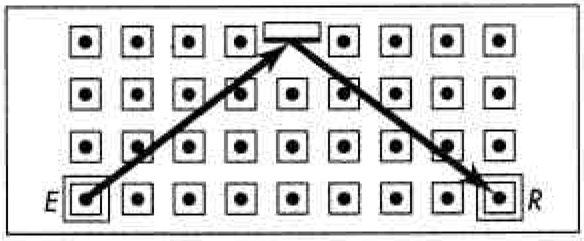
- Key point:
  - The brother remains in a single inertial reference frame, but the sister does not
  - Breaks symmetry of the situation

Spacetime maps

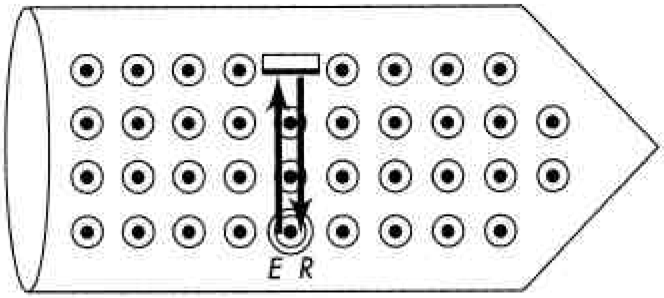
- Plot path of object in space and time
  - Path is called a "world line"
  - Suppress two of the spatial coordinates
- Example:
  - Light pulse bouncing off mirror in rocket
    - Event E = emission of pulse (at origin)
    - Event R = reception of pulse

  - Draw spacetime maps in different reference frames: $c^2t'^2 - x'^2$ always has the same (positive) value ➔ define a curve on the spacetime map

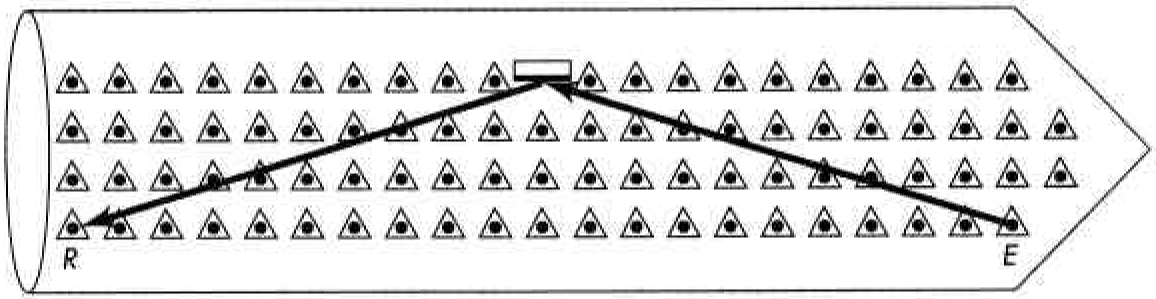
TW Figure 5-2



LABORATORY PLOT

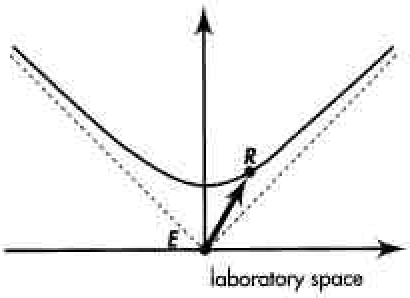


ROCKET PLOT

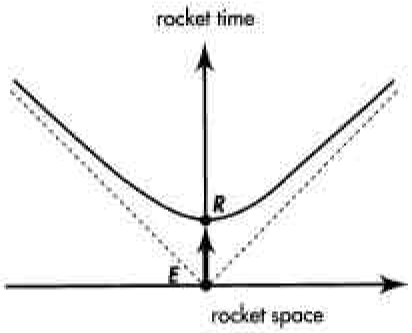


SUPER-ROCKET PLOT

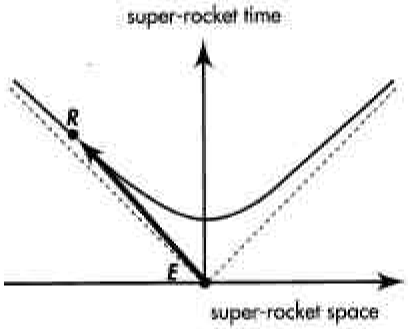
TW Figure 5-3



LABORATORY SPACETIME MAP



ROCKET SPACETIME MAP

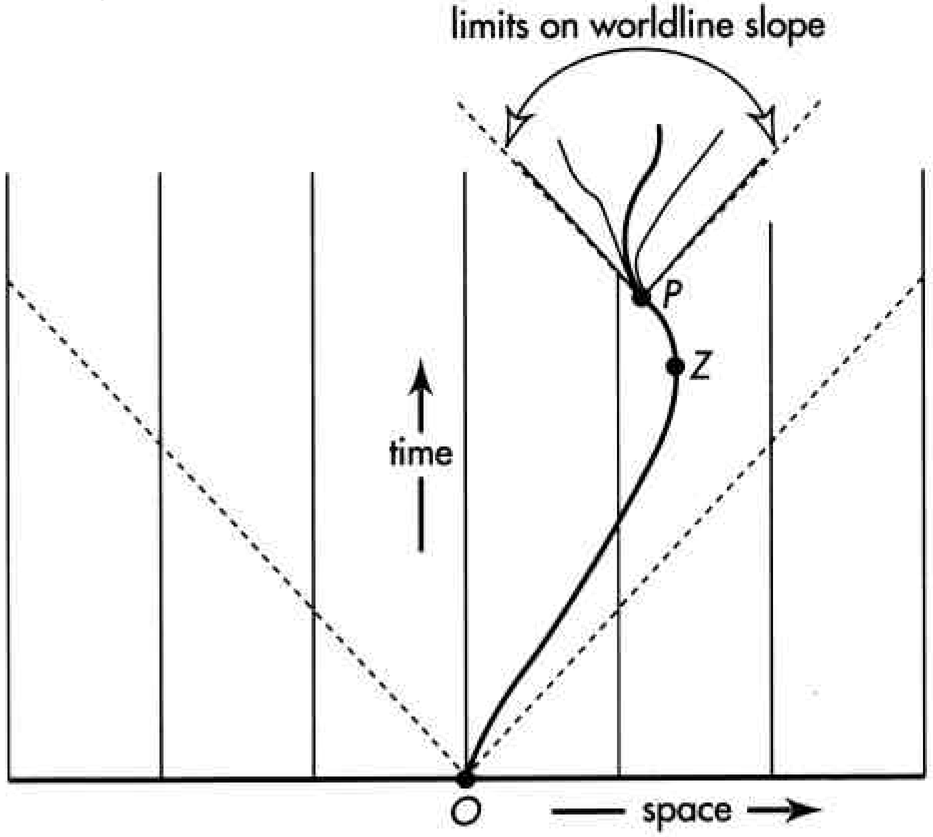


SUPER-ROCKET SPACETIME MAP

Worldline of a moving particle

- Slope must always be greater than unity (or speed will exceed c)

TW Figure 5-6



Proper time for clock moving along a worldline

View a small section of the worldline

- if small enough it will be straight to arbitrary accuracy

$\Delta\tau^2 = c^2\Delta t^2 - \Delta x^2$

$= c^2\Delta t'^2$ in instantaneous rest frame of clock, S'

$\Delta\tau /c = [\Delta t^2 - \Delta x^2/c^2]^{1/2} = \Delta t/\gamma$ is the increase in proper time

Integrate to obtain total proper time

Total proper time elapsed in travelling along a worldline is $\int d\tau /c = \int dt /\gamma$

Length of the worldline

- In ordinary space, straight line has shortest length
- In Lorentz space, straight line has longest proper time
- Contrast

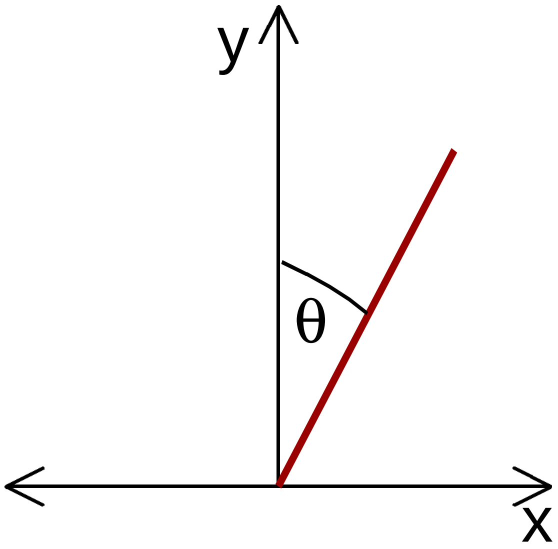
$dl^2 = dx^2+dy^2+dz^2$ (Euclidian space)

with

$d\tau^2 = c^2dt^2 – (dx^2+dy^2+dz^2)$ (Lorentz space)

Euclidian vs. Lorentz geometry

- Euclidian

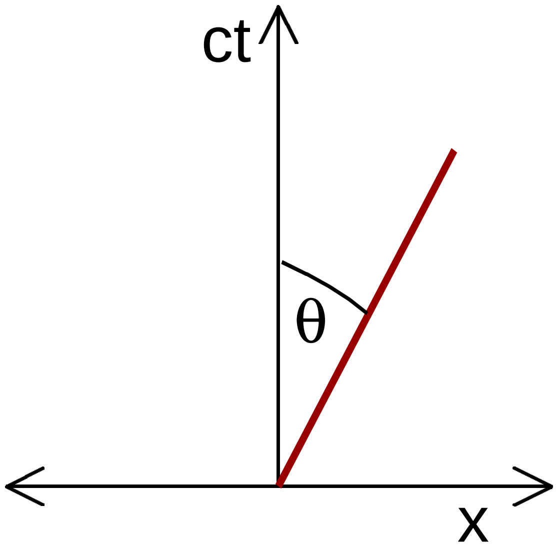


$$\Delta l^2 = \Delta y^2 + \Delta x^2$$

$$= \Delta y^2 (1 + \tan^2\theta)$$

➔ $dl = dy \,/\, |\cos\theta| \geq dy$

- Lorentz



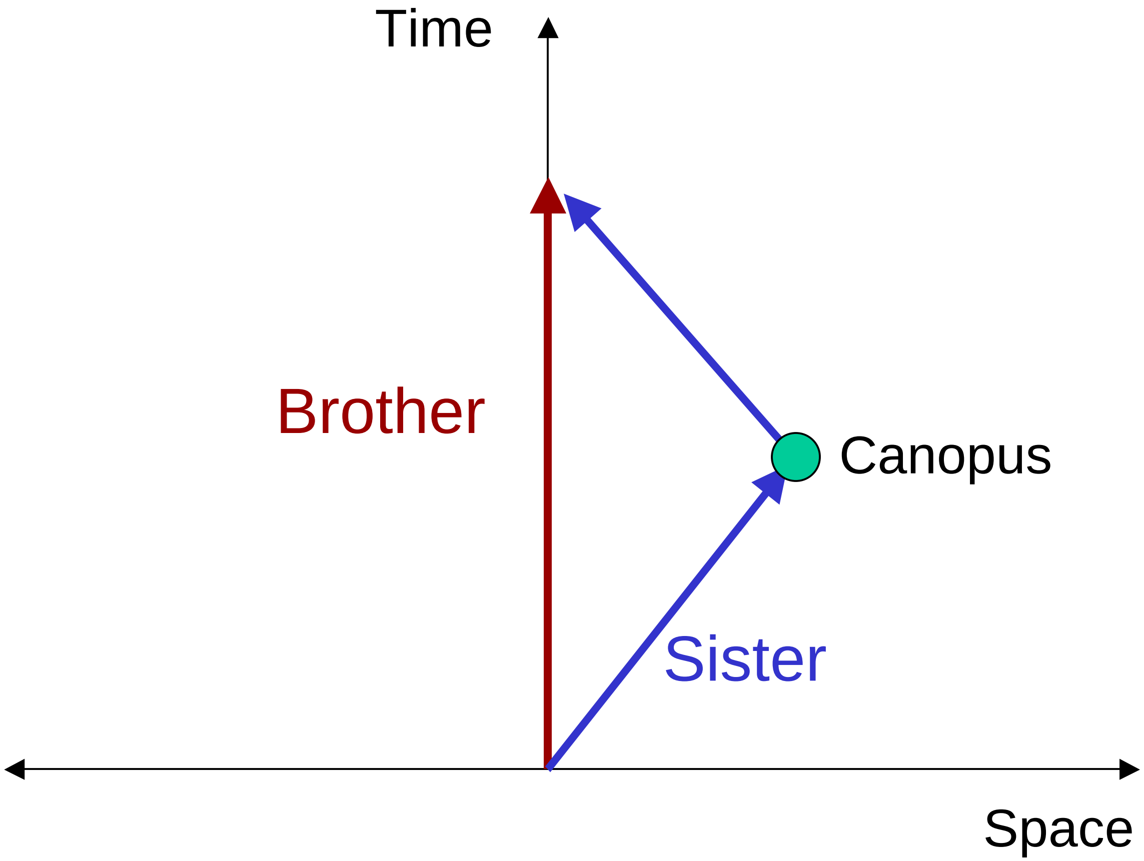
$$\Delta\tau^2 = (c\Delta t)^2 - \Delta x^2$$

$$= (c\Delta t)^2 (1 - \tan^2\theta)$$

➔ $d\tau/c = dt\,(1 - \tan^2\theta)^{1/2} \leq dt$

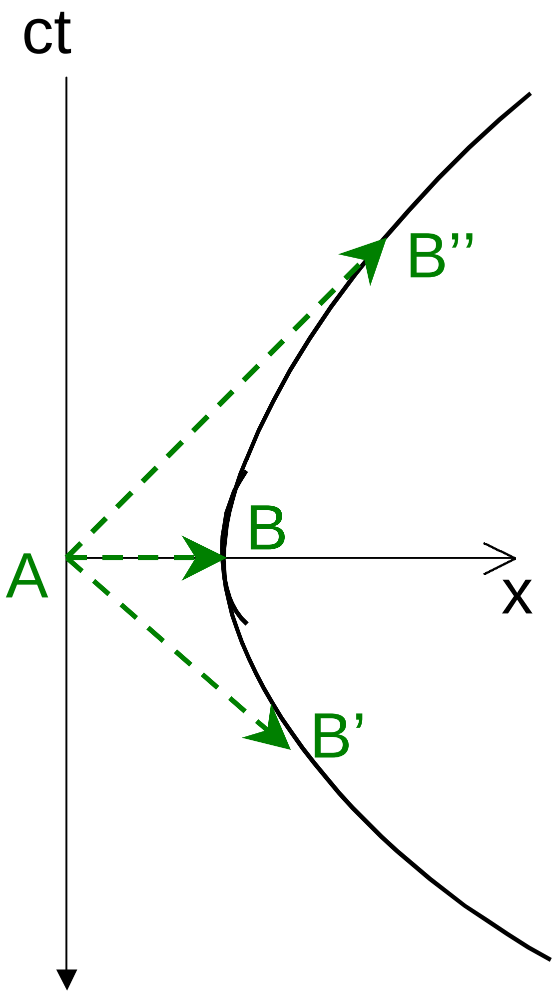
Spacetime map for the twins

- Earth reference frame



Spacelike intervals

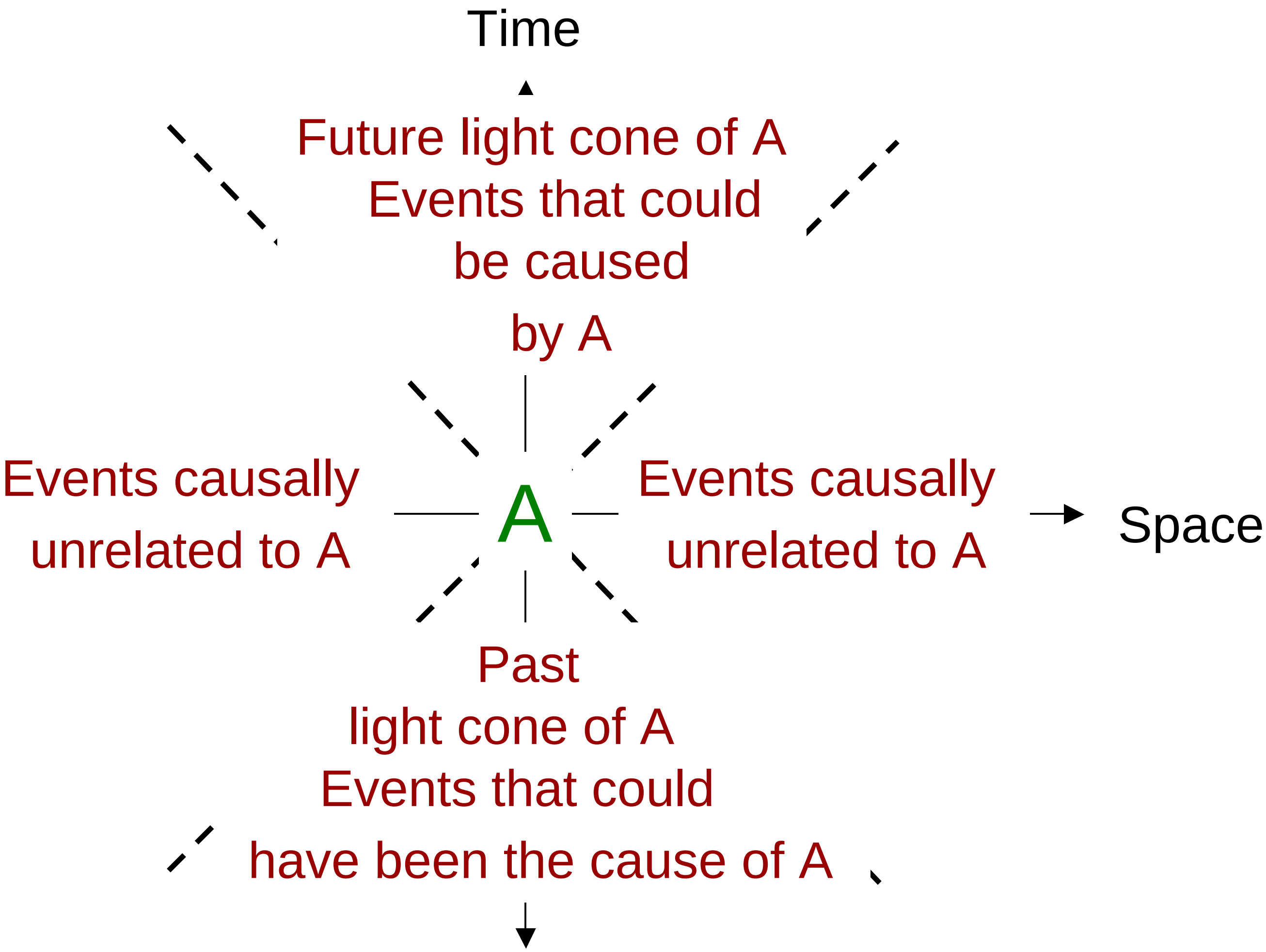
- We previously considered the invariant hyperbola for a timelike interval ($\tau^2 > 0$)
  - – For a *spacelike* interval we have $c^2t^2 - x^2$ constant but negative
  - – Different observers disagree as to whether Event A occurs before or after B
  - – Line joining A to B is *not* the worldline of any real particle



Causality

- If A and B have a spacelike separation, then neither event can be the cause of the other
  - Just as well, since different observers disagree as to which occurred first!
  - Implies that no object (or information-containing signal can move faster than c)
- If A and B have a timelike (or lightlike) separation, all observers agree about which occurred first
  - The earlier event can be the cause of the later event
  - A signal can travel from the earlier to the later event

Regions of spacetime



Time

Future light cone of A
Events that could
be caused
by A

Events causally
unrelated to A

A

Events causally
unrelated to A

Space

Past
light cone of A
Events that could
have been the cause of A

Four-vectors

- Any position in spacetime can be described relative to the origin in vector notation

$$\vec{R} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

An example of a four-vector, $\vec{a} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$

Four-vectors obey a special scalar product rule:

$$\vec{a} \cdot \vec{b} = a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3$$

Transformation of 4-vectors

- Physical quantities that are 4-vectors transform according to the Lorentz transformation

$$a_0 = \gamma a_0' + (\gamma v/c)\, a_1'$$

$$a_1 = (\gamma v/c)\, a_o' + \gamma a_1'$$

$$a_2 = a_2'$$

$$a_3 = a_3'$$

The Invariant “Length” of a 4-vector

- The tranformation rule for 4-vectors implies that they have a “length” that is invariant (same in all IRF)*

Example: for “displacement vector”, $\vec{R}$, we have

$$\text{Length}^2 = \vec{R} \, . \, \vec{R} \quad = (ct)^2 - (x^2 + y^2 + z^2)$$

This is simply the invariant INTERVAL

* analogy: ordinary vectors have a length^2 equal to $(x^2 + y^2 + z^2)$ that is invariant under rotation

L08 →

The velocity 4-vector

- Can we represent additional physical quantities as 4-vectors?

  YES! Consider a particle is moving along its worldline. Define a 4-velocity by

$$\vec{v} = d\vec{R}/d\tau = \lim [\, \Delta\vec{R}/\Delta\tau \,]$$

(4-vector because $\Delta\vec{R}$ transforms as required and $\Delta\tau$ is invariant)

- Motivation: any physics equation written with 4-vectors (e.g. $\vec{a} = \vec{b}$) will be consistent with the first principle of SR

The 4-velocity

- The 4-velocity is given by

$$\vec{v} = (d/d\tau)\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \gamma\,(d/dt)\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \gamma\begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix}$$

- Its direction points along the worldline
- Its length^2 is simply:

$$\vec{v}\,.\,\vec{v} = \gamma^2\,(c^2 - v_x{}^2 - v_y{}^2 - v_z{}^2) = c^2 \quad \text{(INVARIANT!)}$$

The momenergy 4-vector

- Define 4-momentum ("momenergy") as product of "rest mass" and 4-velocity

$$\vec{p} = m_0 \vec{v} = m_0 \gamma \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix} = m \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix}$$

where $m = m_0 \gamma$ is the "relativistic mass"

- Invariant length^2 for 4-momentum

$$\vec{p} . \vec{p} = m_0^2 c^2$$

Momentum

- The “space” part of the 4-momentum vector (components p_1, p_2, p_3) is the regular three momentum, $\underline{p} = m\ \underline{v}$

- Note that the relativistic mass, m, is a function of velocity, $m = m_0\ \gamma$

Energy

- The “time” part of the 4-momentum (component p_0) is the ENERGY/c

  i.e. $E = p_0 c = m c^2 = \gamma m_0 c^2$

- Limit of small v (Newtonian limit)

  $\gamma = (1 - v^2/c^2)^{-1/2} \sim 1 + \frac{1}{2} v^2/c^2$

  $\rightarrow E = m_0 c^2 + \frac{1}{2} m_0 v^2$

  Rest mass energy

  Kinetic energy

Invariant length of momenergy

- Momenergy can be written

$$\vec{p} = \begin{pmatrix} E/c \\ \underline{p} \end{pmatrix}$$

where $\underline{p} = m\underline{v} = \gamma m_0 \underline{v}$

and $E = mc^2 = \gamma m_0 c^2$

- Invariant length^2 = $m_0^2 c^2 = E^2/c^2 - p^2$

i.e. $E^2 - p^2 c^2 = E'^2 - p'^2 c^2$

Conservation of momenergy

- Newtonian Laws of energy and momentum conservation become the law of conservation of 4-momentum in SR

- For an isolated system

Total 4-momentum remains constant:

$d\vec{p}/d\tau = 0$ ➔ $d\vec{p}/dt = 0$ (in any frame)

(Law of physics involving 4-vector ➔ same in any IRF)

L09 ⟶

Mass-energy equivalence

- 0^{th} component of momenergy vector:

  Energy = $\gamma m_0 c^2 = mc^2$

  Would we see an increase in inertia and weight as a result of this γ factor? YES!

Examples:

Inelastic collision ➔ heat ➔ mass increases

Helium atom mass $< 2m_p + 2m_n + 2m_e$

Massless particles

- Photons travel at speed c.

  γ is infinite

  Rest-mass is zero

  Momenergy is finite

  |Momenergy| is zero

- Quantum theory

  Energy, $E = h\nu$

  ➔ Momentum, $p = E/c = h\nu/c$

Lorentz transformation for photons

- Suppose in frame S, the photon 4-momentum is

$$\begin{pmatrix} E/c \\ E/c \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} h\nu/c \\ h\nu/c \\ 0 \\ 0 \end{pmatrix}$$

- Transform to S' frame, moving at speed v in the photon propagation direction:

$$\text{4-momentum} = \begin{pmatrix} h\nu'/c \\ h\nu'/c \\ 0 \\ 0 \end{pmatrix} = \gamma\,(1 - v/c) \begin{pmatrix} h\nu/c \\ h\nu/c \\ 0 \\ 0 \end{pmatrix}$$

Doppler shift

- The change in frequency is given by

$$\nu'/\nu = \gamma(1 - v/c) = (1 - v/c)^{1/2}\,(1 + v/c)^{-1/2}$$

➔ if $v > 0$, $\nu' < \nu$ ➔ redshift

if $v < 0$, $\nu' > \nu$ ➔ blueshift

- Limit $v \ll c$ ➔ $\nu'/\nu \sim 1 - v/c$
- Limit $v \to c$ ➔ $\nu'/\nu \to 0$

L10 →

Particle accelerators

- Accelerate particles close to c, then allow them to collide and create new particles

  If energy of colliding particles is E_{CM} in the center-of-mass frame, new particles with total rest mass energy up to E_{CM} can be created.

$p\bar{p}$ colliders

- Suppose we smash an antiproton of energy $E = \gamma\, m_p c^2$ into a stationary target containing protons (γ up to almost 1000 at the Tevatron)
  - Lab frame: total $\vec{p} = \begin{pmatrix} E/c \\ \underline{p} \end{pmatrix} + \begin{pmatrix} m_p c \\ \underline{0} \end{pmatrix} = \begin{pmatrix} E/c + m_p c \\ \underline{p} \end{pmatrix}$
  - Invariant $\vec{p} \,.\, \vec{p} = (E/c + m_p c)^2 - p^2$
    $= (E/c)^2 + 2Em_p + (m_p c)^2 - p^2 = 2(m_p c)^2 + 2Em_p$
    $= 2(1+\gamma)(m_p c)^2$
- In center of mass frame, energy is

$$E_{CM} = c(\vec{p} \,.\, \vec{p})^{1/2} = [2(\gamma+1)]^{1/2}\, m_p c^2$$

$p\bar{p}$ colliders (2)

- Alternatively, suppose we collide a antiproton particle beam with proton beam traveling in the opposite direction

- In that case the lab frame IS the center of mass frame

  $E_{CM} = 2\,\gamma\, m_p c^2$

  greater by a factor of ~ $(2\gamma)^{1/2}$

Pair production

- Electron-positron pair production can occur via
$\gamma + \gamma \rightarrow e^+ + e^-$
- Suppose the photons are travelling in the positive and negative x-directions:

$$\text{Total momenergy} = \begin{pmatrix} h\nu_1/c \\ h\nu_1/c \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} h\nu_2/c \\ -h\nu_2/c \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} h(\nu_1+\nu_2)/c \\ h(\nu_1-\nu_2)/c \\ 0 \\ 0 \end{pmatrix}$$

$$|\text{Momenergy}|^2 = (h^2/c^2)[(\nu_1+\nu_2)^2 - (\nu_1-\nu_2)^2$$

$$= 4\, h^2\nu_1\nu_2/c^2$$

Pair production (2)

- After the collision, the momenergy in the center-of-mass frame of the e^+e^- pair has $|\text{momenergy}|^2 = (2\,\gamma\, m_e\, c^2)^2 / c^2$
  - – Requirement for pair production:

    $$4\,h^2\nu_1\nu_2/c^2 > 4\,m_e^2\,c^2$$

    $$\Rightarrow [(h\nu_1)(h\nu_2)]^{1/2} > m_e\,c^2$$

  - – Geometric mean of photon energies has to exceed rest-mass energy of electron

General Relativity - overview

- Extends SR by includes the physics of gravity
- Geometric theory of gravity: massive objects cause a curvature of spacetime
- Interval = $\Sigma \Sigma g_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$
- "Field equations" tell us how to compute $g_{\mu\nu}$

Equivalence principle

- Laws of physics the same in any free-falling reference frame.
  - **Free-falling** means that there is no net **non-gravitational** force. Extends our notion of an inertial reference frame.
  - Implies that you cannot tell (from local observations) whether you are in a box free-falling towards Earth or a box floating freely far from Earth's gravity
  - Implies equivalence of gravitational and inertial mass