Numerical methods for physicists

Problem Set 2 – due Friday February 9th

BEFORE YOU START, read www.pha.jhu.edu/~neufeld/numerical/instructions.pdf for useful hints about computer usage AND instructions on how to present your homework.

1) Return to the linear problem considered in Problem Set 1:

\[ A \mathbf{x} = \mathbf{b}, \text{ where } A_{ij} = i + j + 10^n \sin \left[ \exp \left( 3(i + j)/N \right) \right] \]

\[ b_i = 0 \text{ except for } b_1 = 1; \text{ the matrix } A \text{ is } N \times N; \text{ and } n = 3, 2, 1, 0, -1 \ldots. \]

[and i is an integer index, not \( \sqrt{-1} \).]

Now try a singular value decomposition; what does the list of diagonal elements in the “W” matrix tell you about what’s happening? Is the solution you obtain using SVD more meaningful than the one yielded by the LU decomposition method? For small \( w_j \), try zeroing the \( 1/w_j \) elements in the inversion procedure.

2. The light from a star is focused by a large telescope onto the focal plane, and then enters a spectrometer through a rectangular slit. The slit has a 10:1 aspect ratio (i.e. the slit length is ten times the width). Because of the turbulent motions of Earth’s refractive atmosphere, the image of the star is smeared out, such that the intensity of the light at distance \( r \) from the image center is proportional to \( \exp \left[ - \left( r/r_0 \right)^{1.7} \right] \), where \( r_0 \) is a constant.

Use numerical integration – using whatever method you prefer – to determine the fraction of the light, \( f \), that passes through the slit, as a function of \( w/r_0 \), where \( w \) is the slit width. Present your results graphically, showing \( f \) as a function of \( w/r_0 \), for \( w/r_0 \) ranging from 0.01 to 3.

Assume that the stellar image is perfectly centered on the slit.