1. Consider the function

\[ F_n = \frac{x^n \exp[10(x-1)]}{1 + \exp[20(x-1)]} \]

for \( n > 0 \) (for \( n = 0 \), the function has a very sharp peak near \( x = 1 \)). Find the location of the maximum in \( F_n \) for the case \( n = 10 \) by finding the appropriate root of \( F_n' \). First try using a pure Newton-Raphson method; how closely must you bracket the root for this method to succeed? Now try a hybrid bisection-Newton-Raphson algorithm; how closely must you bracket the root in this case? If the initial bracket in the hybrid method is \( x_1 = \varepsilon \ll 1 \) and \( x_2 = 5 \), how many bisection steps are necessary before the Newton-Raphson method gives suggested roots that lie within the bracket?

2. Generate five random (real) numbers, \( x_1 \) through \( x_5 \), between 0 and 1. Multiply out the product \( P = (x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5) \) to obtain a fifth order polynomial: \( P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f \)

Now use Laguerre’s method with deflation to recover the roots of \( P(x) = 0 \) (i.e. to get back the numbers \( x_1 \) through \( x_5 \) that you originally started with).

Graph, on a single plot, the original polynomial \( P(x) \) and the deflated polynomials \( Q(x) \) obtained at each step in the solution. (Added 2/15)

How many iterations is needed to recover each root? How good does your initial guess need to be? (If you start with 0.5 as your initial guess for each root, does the iteration always converge?)