1. Consider the following function

\[ h(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) \quad 0 < t < 16 \]
\[ h(t) = 0 \text{ otherwise} \]

Suppose \( h(t) \) is sampled on a set of 1024 evenly spaced points covering the range \( 0 < t < 16 \).

(a) What is the Nyquist critical frequency?

(b) For the case where \( f_1 = 1 \) and \( f_2 = 5 \), use an appropriate FFT subroutine to find the power spectrum of \( h(t) \):

\[ P(f) = |H(f)|^2 + |H(-f)|^2 \]

where \( H(f) \) is the Fourier transform of \( h(t) \). Plot \( P(f) \) versus \( f \). Can you recover the peaks at \( f = 1 \) and \( 5 \)?

(c) Repeat (b) for the case \( f_1 = 1 \), \( f_2 = 40 \).

Why do you find a peak in the power spectrum around \( f = 23.94 \)?

2. The function \( y(x) \) obeys the differential equation

\[ x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + x^2 y = 0 \]

subject to the boundary conditions \( y(0) = 1 \), \( \frac{dy}{dx}\big|_0 = 0 \). Integrate this equation from \( x = 0 \) to \( x = 2 \) using both the Runge-Kutta and Bulirsch-Stoer techniques. Which one requires fewer function evaluations?