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Observing the fluctuating stripes in high-T_c superconductors

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Abstract – Superfluids and superconductors have been around for a long time and their explanation in terms of the Bogoliubov theory for bosons and the BCS theory for fermions belong to the highlights of twentieth century physics. However, it appears that these theories are too primitive to address the high-T_c superconductivity found in copper oxides. These electron systems seem to behave more like a dense, strongly correlated liquid contrasting markedly with the conventional quantum gasses: these show strong dynamical correlations on mesoscopic length and time scales associated with stripes, a particular form of electron crystallization. Resting on the gauge theory of topological quantum melting in $2+1$ dimensions relevant for the cuprates, we describe the limit which is exactly opposite to the gas limit: the superconductor with the maximum possible amount of transient translational order. We predict that in this "orderly limit" an extra collective mode appears, and this "massive shear photon" can be regarded as a universal fingerprint of the fluctuating stripes. This mode is visible in the electrodynamic response and the ramification of our theory is that electron energy loss spectroscopy can be employed to prove or disprove the existence of dynamical stripes in cuprate superconductors.

It is by now well established [1–3] that the superconducting liquid found in the cuprates is in competition with the so-called stripe phase [4], which can be viewed as a special kind of electron crystal. By studying the behavior of the spin [1,5,6] and phonon [7] system with inelastic neutron scattering an intriguing case emerged that these stripes survive in the form of transient correlations in the liquid up to rather large length ($\sim 10\,\text{nm}$) and time ($\sim 10\,\text{meV}$) scales. This affair is still quite controversial because the data can also be explained in other ways [8–10] and there is a need for a unique, experimentally accessible signature for the "quantum stripes" [11]. In this letter we present such a feature which we identified by studying the quantum field theory describing the topological quantum melting of crystals into quantum (zero temperature) liquid crystals of the smectic and nematic kind [12–15]. In a recent publication we illustrated the powers of this "elastic gauge theory" by deriving the remarkably intricate quantum hydrodynamical mode structure associated with the superfluid quantum smectic state [15]. Here we take these matters one step further: dealing with electron matter, the only meaningful quantum hydrodynamical questions one can ask relate to the electromagnetic response and we analyze here the electromagnetically gauged extension of the theory. By analyzing the charged quantum nematic we arrive at the conclusion that when the transient translational order in the superconducting liquid has a sufficiently long (but finite) range, its electromagnetic response is characterized by an *additional propagating mode* besides the usual plasmon (see fig. 1). We suggest to call this mode the "massive shear photon", since it is just the Higgsed photon associated with the dual (stress) superconductivity carried by the quantum dislocations. This massive shear photon carries electromagnetic strength and we claim that its detection is within reach of present-day experimental techniques, being either elecron energy loss measurements or the next generation of resonant inelastic X-ray scattering machines.

One can already anticipate the presence of the extra mode by a simple quantum hydrodynamical consideration.

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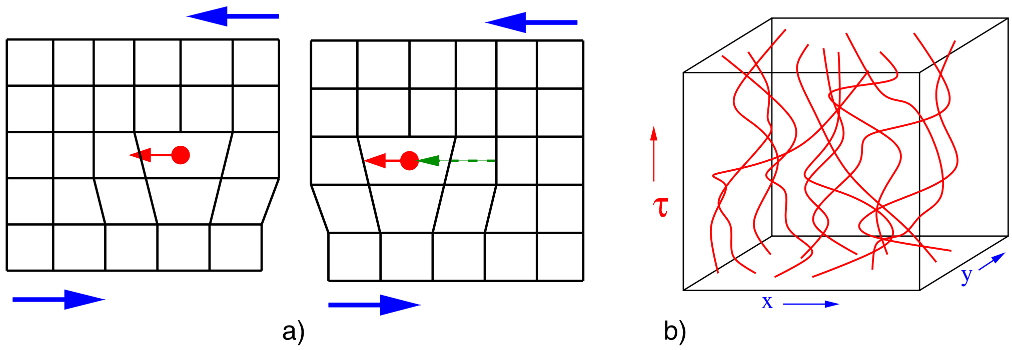

Fig. 1: Role of the dislocations in the destruction of the crystalline order: a) The disclinations, as the topological defects of the ordered crystalline state, are uniquely associated with the shear stress. A dislocation defect glides in order to nullify any shear stress exerted on the solid. Conversely, a tangle of delocalized dislocation screens the shear rigidity at lengths larger than λ_S. b) A single dislocation can only glide [25] (move parallel to its Burgers vector) and its worldline lies entirely in the plane spanned by the time direction and the Burgers vector. When the Burgers vectors are disordered, this constraint effect is averaged over all possible directions.

Shear rigidity, the ability of the medium to respond reactively on a force acting in opposite directions on opposite sides of the medium, is uniquely associated with *translational* symmetry breaking, *i.e.*, crystalline order. In a classical fluid one finds a viscous response on shear, but in the dissipationless superfluids there is no response to shear at all. Superfluids carry only compressional rigidity which in superconductors is manifest in magnetic (Meissner) and electrical screening [16]. Let us assert that a "nearly ordered" superconductor exists which behaves as a solid at short distances, meaning that it has the capacity to mediate shear forces, whereas it loses this capacity at long distances where it turns into a standard superconductor. In analogy with the London penetration depth (of a BCS superconductor) associated with magnetic forces, there should be a characteristic length in the "nearly ordered" superconductor over which shear sources can make their presence felt in the medium, and this length is defined as the "shear penetration depth" λ_S [17]. The natural velocity scale of reactive shear corresponds to the transversal phonon velocity of the (electron) crystal c_T. Assuming a coherent quantum dynamics, one anticipates the presence of "photon-like" excitation in the system carrying the shear stress. This excitation —"the massive shear photon"— is characterized by an energy gap at zero momentum, $\Omega = c_T/\lambda_S$. Since such a mode does not exist in a gaseous system, there is apparently an additional condition for the above argument to become valid, which is: the shear penetration depth should be large compared to the lattice constant $\lambda_S \gg a$.

When this condition is satisfied, a controlled mathematical description is available [12–15], based on modern developments in quantum field theory [18,19], making it possible to arrive at a detailed, quantitative prediction for an observable quantity: the massive shear photon should give rise to a pole in the electron-energy loss function at small momenta and energies (fig. 2), revealing its identity through the characteristic dependence of its pole strength on momentum (fig. 3).

The applicability of the field theory requires some other conditions to be satisfied which are in fact implied by the assumption of a large crystalline correlation length. This means that the system has to be close to a continuous quantum phase transition between the crystal and the superconductor, and this implies in turn: a) The liquid and the solid have to be microscopically similar and since the superconductor is a bosonic entity the crystal is also formed from bosons. It is about preformed pairs which can either form stripes or the superconductor [20], and the theory is dealing with the competition between these two collective states. b) The transition from the crystal to the superconductor is first order, and to make possible a large shear length it appears necessary to assume that the superconductor is at the same time a quantum liquid crystal of either the smectic or nematic kind [21]. It turns out that with regard to the shear stress photon it does not make much of a difference [14] and we will present here results for the simpler nematic superconductor.

The theory can be viewed as the quantum generalization of the famous Nelson-Halperin-Young theory [22] of classical melting in two dimensions, based on the notion that the liquid crystal can be viewed as a crystal where topological defects (dislocations) have proliferated. The quantum version rests on a Kramers-Wannier duality arising in elasticity theory, first explored by Kleinert in the 1980s in the context of the classical problem in three dimensions [18], and only recently implemented on the quantum level [12]. This earlier work contained some technical flaws (ignoring the relativistic character of the dislocation condensate, a faulty gauge fix) obscuring the view on the electrodynamic response. In this letter, we present a summary of the main steps of the correct derivation, and we refer for further details to refs. [14,15,23].

In the "orderly" limit $\lambda_S \gg a$, the constituent bosons of the gaseous limit are no longer relevant and instead the work is done by the collective degrees of freedom: the phonons and the topological defects. The latter correspond to dislocations and disclinations, restoring the translational and rotational symmetry, respectively. In this language, liquid crystals are states where dislocations have proliferated, restoring translations, while disclinations are still massive excitations such that rotational symmetry remains broken [15,19].

The starting point is the Lagrangian describing the collective properties (phonons, and topological defects) of a ("Wigner") crystal of charged bosons in 2+1 Euclidean space-time dimensions. It follows from the gradient expansion [14,18] in terms of the displacement fields u^a,

$$\mathscr{L}_0 = \frac{1}{2}\partial_\mu u^a C_{\mu\nu ab}\partial_\nu u^b + i\mathcal{A}^a_\mu \partial_\mu u^a + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where $C_{\mu\nu ab}$ is a short hand for the tensor of elastic moduli, now including the kinetic energy $(\rho/2)(\partial_\tau u^a)^2$ (Greek indices label space-imaginary time directions $\sim x, y, \tau$, Latin indices space directions $\sim x, y$; notice that $C_{\tau\tau ab} = \rho\delta^{ab}$, where ρ is the mass density). In a companion long paper [23], we will present a detailed derivation, accounting for a general elasticity tensor, paying particular attention to the rectangular 2D lattice of relevance to cuprates. Since the results are qualitatively the same regardless of the lattice type, we limit ourselves here to isotropic elasticity characterized by just a shear and compression modulus, μ and κ, respectively, related by the Poisson ratio $\kappa = \mu(1+\nu)/(1-\nu)$. This defines the velocities $c_T = \sqrt{\mu/\rho}$, $c_L = c_T\sqrt{2/(1-\nu)}$, $c_\kappa = c_T\sqrt{(1+\nu)/(1-\nu)}$, corresponding to the transversal and longitudinal phonon velocities of the crystal, while c_κ is the compressional (real sound) velocity. The last term in eq. (1) is the usual Maxwell term describing the dynamics of electromagnetic fields. The electromagnetic vector potentials (A_μ) couple to the elastic strains via the effective combination [12] $\mathcal{A}^a_\mu = (n_e e^*)\,[A_\tau\delta_{\mu a} - A_a\delta_{\tau\mu}]$ (e^* is the microscopic electrical charge, n_e is the density of charged particles). The scales associated with the electromagnetism are the plasmon frequency $\omega_p = \sqrt{n_e(e^*)^2/\rho}$, the Debye electrical screening length $\lambda_e = c_T/\omega_p$, and the Debye momentum $q_e = 1/\lambda_e$.

The crucial insight, due to Kleinert, is to turn this into a "stress gauge theory" [18]. The first step is to use strain-stress duality, such that eq. (1) becomes the sum of the following two Lagrangians:

$$\mathscr{L}_{dual,0} = \frac{1}{2}\left[\frac{(\sigma^a_a)^2}{\kappa} + \frac{(\sigma^x_x - \sigma^y_y)^2 + (\sigma^x_y + \sigma^y_x)^2}{4\mu} + \frac{(\sigma^a_\tau)^2}{\rho}\right], \quad (2)$$

$$\mathscr{L}_{dual,EM} = i\mathcal{A}^a_\mu C^{-1}_{\mu\nu ab}\sigma^a_\mu + \frac{1}{2}\mathcal{A}^a_\mu C^{-1}_{\mu\nu ab}\mathcal{A}^b_\nu + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (3)$$

We have written $\mathcal{L}_{dual,0}$, the Lagrangian of isotropic elasticity, explicitly in terms of the stress fields σ^a_μ, with the σ^a_τ having the status of canonical momenta.

Before addressing the electromagnetism, let us first review briefly the elastic sector, eq. (2), of the above theory [12–15]. The key to recognizing the gauge theoretical nature of the problem is the realization that stress is conserved, $\partial_\mu\sigma^a_\mu = 0$, and this can be imposed in 2+1D by expressing the stresses in terms of stress gauge fields,

$$\sigma^a_\mu = \epsilon_{\mu\nu\rho}\partial_\nu B^a_\rho. \quad (4)$$

where the B^a_ρ are "flavored" non-compact $U(1)$ fields, in fact subjected to the extra constraint associated with the vanishing of the antisymmetric components of the spatial stress, $\sigma^x_y - \sigma^y_x = 0$ (Ehrenfest constraint). The magic of these stress-gauge fields is, that like in vortex duality [24], the sources of these stress gauge fields correspond to the non-integrable displacement field configurations [18],

$$\mathscr{L}_{disl} = iB^a_\mu J^a_\mu, \qquad J^a_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu\partial_\lambda u^a. \quad (5)$$

The fields J^a_μ correspond to the dislocation currents. These can be factorized as $J^a_\mu = b^a \mathscr{J}_\mu$: worldlines of dislocations with Burgers vector $\mathbf{b} = (b_x, b_y)$. This implies in turn that $B^a_\mu J^a_\mu \to (b^a B^a_\mu)\mathscr{J}_\mu$. Hence, we arrive at a description of elasticity which is a close sibling of electromagnetism: the stress gauge fields ("stress photons") express the capacity of the elastic medium to propagate elastic forces, and the dislocations take the role of charged sources. Using the Coulomb gauge fix [24], it is easy to deduce that of the three physical photons (four gauge degrees of freedom, plus the Ehrenfest constraint) two are propagating, corresponding to the transversal and longitudinal phonon, and the third one mediates "Coulomb" interactions between static dislocations. To complete this short exhibition, dislocations have the peculiar property that they can only propagate in the direction of their Burgers vectors [25]. This turns out to have the ramification that the spatial components of the dislocation current tensors have to be symmetric ($J^x_y = J^y_x$), which in turn has the consequence that dislocations only act as sources of shear stress (both the longitudinal and transversal stress components: $\sigma^x_x - \sigma^y_y$ and $\sigma^x_y + \sigma^y_x$, respectively), decoupling completely from compressional stress $\sigma^x_x + \sigma^y_y$.

In the $2+1$ space-time dimensions of relevance to cuprates, the miracle is that the quantum-nematic crystal can be equally well viewed as a "dual superconductor" [12,13], in close analogy with the superfluid-superconductor duality in $(2+1)$-dimensional phase dynamics [24,26,27]. In the path-integral language, the "dual nematic shear superconductor" corresponds to a tangle of dislocation worldlines in space-time. As we explained elsewhere [15], these describe either smectic or nematic quantum liquid crystals depending on the orientational ordering of the Burgers vectors: in the smectic, dislocations condense with their Burgers vectors oriented in one particular direction while in the nematic they condense with equal probability for their Burgers vectors to be oriented along all allowed directions [19][1].

[1]The quantum hexatic has a D_6 point group symmetry which we do not explicitly invoke here. The precise lattice structure is irrelevant (in an RG sense) for the character of the asymptotic

The dislocation condensate is gauged by the stress gauge fields, and eq. (5) implies a covariant derivative structure of the form $|(\partial_\mu - ib^a B^a_\mu)\Psi|^2$, where Ψ is the order parameter of the dislocation condensate; as we overlooked in an earlier work [12], it is vital to keep here the time derivatives given the relativistic nature of the dislocation condensate [24]. Elsewhere, we present in detail the averaging procedure leading to the correct Higgs term in the stress sector [14,23]. The "bare" Higgs term (without the Ehrenfest and glide [25] constraint imposed) for the quantum hexatic is $\mathscr{L}_{H,bare} = \frac{1}{4}|\Psi_0|^2 B^a_\mu B^a_\mu$, where Ψ_0 is the expectation value of the disorder field. The glide constraint is implemented via a Lagrange multiplier term $\mathscr{L}_{glide} = i\lambda(J^y_x - J^x_y)$. This operation removes the compressional stress $\sigma^x_x + \sigma^y_y$ from the Higgs term, having also the effect of turning [12,14,23] the second sound velocity of the condensate into the "glide velocity" $c_g = c_T/\sqrt{2}$. Subsequently, the Ehrenfest constraint is implemented by eliminating the antisymmetric stresses $\sigma^y_x - \sigma^x_y$ from the Higgs term. It is particularly convenient to represent the resulting Higgs term in a gauge invariant way [13],

$$\mathscr{L}_H = \frac{1}{2}\frac{\Omega^2}{2\mu}\left[\frac{(\sigma^x_x - \sigma^y_y)^2}{(\partial_\tau)^2 + c_g^2(\partial_x)^2} + \frac{(\sigma^x_y + \sigma^y_x)^2(2 + c_T^2(\partial_x)^2/(\partial_\tau)^2)}{(\partial_\tau)^2 + c_T^2(\partial_x)^2}\right], \tag{6}$$

where we defined the "shear Higgs mass" as $\Omega = |\Psi_0|\sqrt{\mu}$. The derivatives in denominators correspond to momentum operators in real space (*i.e.*, Fourier transform of $1/q^2$, etc.).

This result is revealing: by counting derivatives it is immediately obvious that both the longitudinal $(\sigma^x_x - \sigma^y_y)$ and transversal $(\sigma^y_x + \sigma^x_y)$ shear stresses acquire a Higgs mass Ω, while compressional stress is left unaffected. However, in the longitudinal sector, due to the (dynamical) glide constraint and the effect of the averaging over the Burgers directions, the velocity of dislocations is reduced by a factor: $c_g = c_T/\sqrt{2}$.

Let us now turn to the subject of the present paper: the electrically charged systems and their electromagnetic response. According to standard linear response theory, the electrodynamic response is governed by the dielectric tensor, parametrized in terms of the transversal and longitudinal dielectric functions. For isotropic media, these can be expressed in terms of the diagonal elements of the photon self-energy Π as [28] (with q the momentum and ω_n the Matsubara frequencies),

$$\hat{\varepsilon}_L(q,\omega_n) = 1 - \frac{\Pi_\tau(q,\omega_n)}{q^2}, \tag{7}$$

$$\hat{\varepsilon}_T(q,\omega_n) = 1 - \frac{\Pi_T(q,\omega_n)}{\omega_n^2}, \tag{8}$$

in the Coulomb gauge for the EM fields, $\partial_a A_a = 0$. Our task is to determine the Π's in the nematic dislocation condensate.

We observe that the total dual action eqs. (2), (3) and (6) can schematically be rewritten as

$$\mathscr{L}_{dual} = \frac{1}{2}B^a_\mu(\mathcal{G})^{-1}_{\mu\nu ab}B^b_\nu + iB^a_\mu g_{a,\mu\nu}A_\nu + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}A_\mu(\Pi^{bare})_{\mu\nu}A_\nu, \tag{9}$$

where all the physics of the stress sector is lumped together in the fully dressed stress gauge field propagator $(\mathcal{G})_{\mu\nu ab}$. The stress gauge fields couple to the EM fields via the coupling constants $g_{a,\mu\nu}$ and, remarkably, the strain-stress duality followed by the parametrization in terms of the stress photons leads automatically to a Meissner-like term $\sim A_\mu(\Pi^{bare})_{\mu\nu}A_\nu$. The coupling constants $g_{a,\mu\nu}$ and the Π^{bare} are tabulated in ref. [14].

Given that the coupling between stress and EM gauge fields is linear, the photon self-energies are easily derived by straightforwardly integrating out the stress photons from eq. (9),

$$-\Pi_{\mu\nu}(q,\omega_n) = -\Pi^{bare}_{\mu\nu} - g^*_{a,\kappa,\mu}(\mathcal{G}(q,\omega_n))_{\kappa\kappa' ab}\, g_{b,\kappa',\nu}. \tag{10}$$

One can show that the above does reproduce correctly the electrodynamics of the Wigner crystal, obtained by ignoring the dislocations all together. As it turns out, the longitudinal components of the "pseudo-Meissner" term Π^{bare} takes care of electrical screening, and the fact that the phonons acquire a plasmon gap. However, in the transversal magnetic sector, it is "eaten" by counter terms coming from the integrations over the stress gauge fields showing that the crystal does not support diamagnetic (Meissner) screening.

As we already noticed in an earlier work [12], from the transverse electromagnetic response it follows that the quantum hexatic is also an electromagnetic Meissner state. This has the peculiar ramification that off-diagonal long-range order (ODLRO) involving the constituent bosons is apparently not a necessary condition for superconductivity, since these bosons do not even exist in this limit! ODLRO associated with the disorder operators (the dislocations) suffices.

The explicit expressions for the fully dressed stress gauge field propagator $\mathcal{G}$ in eq. (9), now incorporating the shear Higgs mass and condensate dynamics, are obtained by reinserting the stress gauge fields in eq. (6). Using eqs. (8), (10) the dielectric functions follow from a lengthy but straightforward calculation. We find that the Coulomb photon A_τ and the "true" (radiative) photon A_T each acquire self-energies given by

$$-\Pi_\tau = q^2\frac{\omega_p^2(\omega_n^2 + c_g^2q^2 + \Omega^2)}{(\omega_n^2 + c_L^2q^2)(\omega_n^2 + c_g^2q^2) + \Omega^2(\omega_n^2 + c_K^2q^2)}, \tag{11}$$

long-wavelength behavior. The isotropic continuum theory equally emulates the long-wavelength behavior in the presence of D_6 or other point group symmetries. Continuum isotropic elasticity qualitatively describes many materials quite well regardless of their specific point group symmetries. Anisotropic lattice effects are interesting but do not fundamentally alter the character of the response functions.

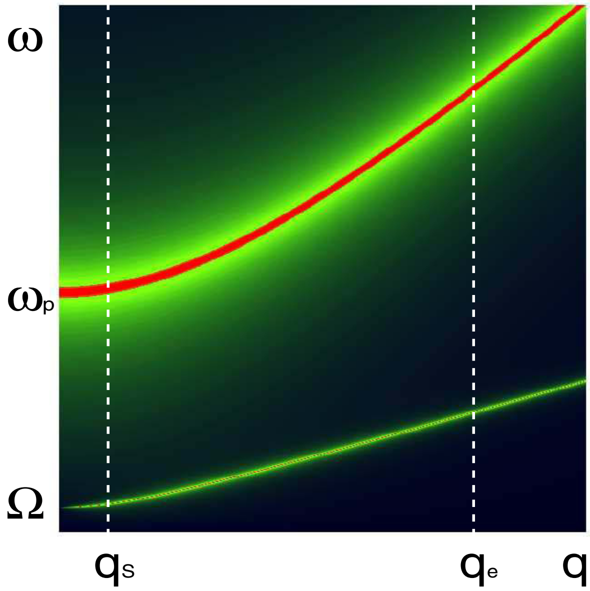

Fig. 2: The electromagnetic absorptions in the "nearly ordered" superconductor as seen by electron-energy loss spectroscopy ($\mathrm{Im}[1/\varepsilon_L(q,\omega)]$) as a function of frequency ω and momentum q. In this example, we take a typical Poisson ratio $\nu = 0.28$, characterizing the "background" Wigner crystal, while the "shear Higgs mass" $\Omega = 0.05\omega_p$ and shear penetration depth $\lambda_S = 1/q_S = c_T/\Omega$ are taken to be representative for the situation in cuprate superconductors ($\omega_p \simeq 1\,\mathrm{eV}$, $\Omega \simeq 50\,\mathrm{meV}$, $\lambda_S \simeq 10\,\mathrm{nm}$). Besides the strong plasmon pole (set by ω_p) dominating the long-wavelength dielectric response, we also find a weak absorption which corresponds to the massive shear photon (set by the "shear Higgs mass" Ω) giving away the presence of the "dual shear superconductor". This is to be regarded as the unique fingerprint of a superconductor characterized by transient translational order extending over distances large compared to the lattice constant.

$$-\Pi_T = \omega_p^2 \frac{\omega_n^2(\omega_n^2 + c_T^2 q^2) + \Omega^2(\omega_n^2 + c_g^2 q^2)}{(\omega_n^2 + c_T^2 q^2)(\omega_n^2 + c_T^2 q^2) + \Omega^2(\omega_n^2 + c_g^2 q^2)}. \tag{12}$$

and we find for the dielectric functions,

$$\hat{\varepsilon}_L = 1 + \frac{\omega_p^2(\omega_n^2 + c_g^2 q^2 + \Omega^2)}{(\omega_n^2 + c_L^2 q^2)(\omega_n^2 + c_g^2 q^2) + \Omega^2(\omega_n^2 + c_K^2 q^2)}, \tag{13}$$

$$\hat{\varepsilon}_T = 1 + \frac{\omega_p^2}{\omega_n^2} \frac{\omega_n^2(\omega_n^2 + c_T^2 q^2) + \Omega^2(\omega_n^2 + c_g^2 q^2)}{(\omega_n^2 + c_T^2 q^2)(\omega_n^2 + c_T^2 q^2) + \Omega^2(\omega_n^2 + c_g^2 q^2)}. \tag{14}$$

As we already discussed, the static limit of the transversal (magnetic) response reveals that we are dealing with a Meissner state. The differences between "gaseous" and "orderly" superconductors show up, however, at finite frequencies and momenta and there are no experiments available that can probe the transversal response in the relevant kinematical regime. This is different for the *longitudinal* EM response eq. (13), that can be "routinely" measured through the electron energy loss function $\mathrm{Im}[1/\varepsilon_L(q,\omega)]$. In fig. 2 we show an example of the loss spectrum, as calculated from eq. (13), using parameters that might well be representative for the situation in the cuprates when fluctuating stripes do exist. Generically, the "order" superconductivity reveals itself in the longitudinal electromagnetic absorptions through the existence of a second propagating mode at long wavelength, besides the ubiquitous plasmon pole. This is the "massive shear photon" we discussed in the beginning, which turns out to acquire a small but finite electromagnetic strength making it visible in the longitudinal electromagnetic response.

How to think about the various parameters in the theory? The stage is set by the standard electrodynamical parameters which are well known in cuprate superconductors: the (ab-plane) plasmon frequency $\omega_p \simeq 1\,\mathrm{eV}$ while the charge (plasmon) velocity is of order $1\,\mathrm{eV\AA}$ according to EELS measurements [29], while the electrical screening length $1/q_e$ is of order of the lattice constant. Surely, our quantum hydrodynamical theory knows nothing about microscopic dynamics and, henceforth, it has nothing to say about the kinematical regime at large momenta $\sim q_e$. When fluctuating stripes do exist, they should be characterized by an electronic shear velocity scale c_T being of order of the compressional plasmon velocity since poisson ratio's are always of order one. Henceforth, our theory needs a single unknown parameter: the shear Higgs mass Ω, or equivalently, the inverse shear penetration depth q_S. Resting on the stripe interpretations of the spin fluctuations seen by neutron scattering, the characteristic disorder scales in the spin system should be rooted in the charge fluctuations discussed here: the shear Higgs mass Ω should be small ($\sim 10^{-2}\,\mathrm{eV}$) [5,6], while the characteristic length scale parametrized by λ_S should be $\sim 10\,\mathrm{nm}$, consistent with a typical electronic velocity $c_T \simeq 1\,\mathrm{eV\AA}$. Figure 2 is a typical sample for this parameter regime.

The weight of the shear photon has a non-trivial dependence on both the ratio $\theta = \Omega/\omega_p$ as well as on the momentum q. This is easily computed from eq. (13) and the results are summarized in fig. 3. In the "hydrodynamical" long-wavelength regime $q < q_S$, the weight of the shear photon $I_{sh}(q) = \frac{\theta(1-\nu)}{4(1-\theta^2)} q^2 + O(q^4)$. I_{sh} has a maximum at intermediate momenta $q_{max} = \sqrt{2(1-\nu)/(3(3+\nu))} \sim 0.4 q_e$ where the shear photon acquires a weight relative to the plasmon $(I_{sh}(q_{max})/I_p) \simeq (3\sqrt{3(1-\nu)})/(16\sqrt{3+\nu})\,\theta^2$ which we expect to be of order 0.1% in cuprates given that $\theta \simeq 0.05$: the electromagnetic weight of the shear photon is expected to be very small. This momentum and θ-dependence of the electromagnetic strength of the shear photon can however be taken as its fingerprint, since it is rooted in the mechanism through which the shear photon becomes electromagnetically active. Both shear stress and dislocations forming the building blocks for the massive shear photon do not carry volume and are therefore electrically neutral. However, in the Wigner crystal and in the liquid, at distances small compared to

Fig. 3: Pole strengths of the plasmon (red) and the novel, shear, photon (yellow) in the electron energy loss spectrum: the scale is adjusted to accommodate faint features of the shear photon. Two wavelength regimes have different behaviour: at intermediate wavelengths ($q_S < q < q_e$) the pole develops the absolute maximum of order of a tenth of θ^2, *i.e.* 0.1% for the gap estimates in the text; at long wavelengths ($q < q_S$) the strength grows with the square of the wave number (see text).

λ_S, the plasmon carries like longitudinal phonon shear components at finite momenta. These shear components have to be "removed" from the plasmon, turning it into the purely compressional mode of the liquid at distances large compared to λ_s. This causes a linear mode coupling between the plasmon and the shear photon, with the latter "stealing" some electromagnetic weight from the former. This explains the strong dependence on θ, the characteristic maximum, and the vanishing of the weight at $q \to 0$ as even in the Wigner crystal, the plasmon turns into a purely compressional mode in the long-wavelength limit.

Is the shear photon measurable? A first possibility is Raman. In fact, it might well be that it is related to the A_{1g} electronic peak, argued by Uemura [30] to be related to a "roton" and the shear photon is "roton-like". However, to the best of our understanding, there is only Raman activity in the two-shear-photon channel which renders it to be rather indirect. Amusingly, Uemura's observation [30] that the superconducting transition temperature $k_B T_c$ scales with a dynamical frequency scale finds a natural explanation in our "orderly superconductor": T_c is set by the transition temperature of the dual shear superconductor and since this is a truely relativistic (Higgs) condensate, T_c is set by the Higgs mass and therefore $k_B T_c \simeq \hbar\Omega$!

Obviously, the easy way is to measure directly the loss function in this kinematical regime. This requires a millielectron volt energy, and nanometer spatial resolution. Although such instruments are not available right now it just seems to involve a relatively minor engineering effort to get into this regime: reflection (low energy) EELS has the resolution but for unknown reasons there are problems measuring the electronic response [31]; transmission (high energy) EELS is limited to a resolution of order of ~ 100 meV but there seems much room for improvement using modern electron optics. The strongest contender appears to be the resonant inelastic soft X-ray scattering where at present a major instrumental development program is unfolding aimed at reaching meV resolution.

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