1. **Photometric parallax (3 points).** Use the SDSS Navigate tool to determine the $g - r$ and $r - i$ colors of the star at coordinates 10h39m46.00s +64d31m20.6s. Approximately what stellar type is this star? Can you estimate its mass? Can you estimate its distance? Provide references.

2. **Malmquist bias (3 points).** (3.4 from BM) Evaluate the ratio of the mean luminosities of (a) a mag-limited sample and (b) a volume-limited sample of objects with luminosity function given by equation

$$\Phi(M) = \frac{\Phi_0}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{(M - M_0)^2}{2\sigma^2}\right).$$  

Assume that the population is homogeneously distributed in space. If both surveys are infinitely deep, are the mean luminosities the same or different from one another?

3. **Surface brightness and inclination (4 points).** (1.4 of BT) An axisymmetric transparent galaxy has luminosity density that is constant on spheroids $R^2 + z^2/q^2$ having axis ratio $q$. A distant observer located on the symmetry axis of the galaxy sees an image with circular isophotes and central surface brightness $I_n$. A second distant observer, observing the galaxy from a line of sight that is inclined by an angle $i$ to the symmetry axis, sees an image with elliptical isophotes with axis ratio $Q < 1$ and central surface brightness $I_0$.

(a) What is the relation between $I_0$, $I_n$ and $Q$? Hint: the answers are different for oblate ($q < 1$) and prolate ($q > 1$) galaxies.

(b) What is the relation between $q$, $Q$ and $i$?

(c) Assuming that galaxies are oriented randomly, what fraction are seen from a line of sight that lies within 10 deg of the symmetry axis? From within 10 deg of the equatorial plane?

4. **Potential and brightness profile of a globular cluster (2+3+2+1 points).**

(a) (2.9 from BT) If a transparent, spherical stellar system has constant mass-to-light ratio $\gamma$, prove that the central potential is (Ciotti 1991)

$$\Phi(0) = -4G\gamma \int_0^\infty dR I(R),$$  

where $I(R)$ is the surface brightness at projected radius $R$.

(b) The surface brightness distribution of many globular clusters is described by the Plummer law:

$$I(R) = \frac{I_0}{(1 + R^2/b^2)^2}.$$  

Assuming that the cluster is spherically symmetric, use the result from (a) to calculate the escape velocity from the center of such globular cluster. Evaluate your result (in km/sec) for globular cluster M999 with central bolometric surface brightness $I_0 = 16$ mag/arcsec$^2$, core size $b = 3$ pc and mass-to-light ratio $\gamma = 2M_\odot/L_\odot$. (Hint: you need to translate mag/arcsec$^2$ into some more reasonable units first.)

(c) What is the luminosity density distribution for the surface brightness profile given by equation (3)?

(d) What is the total mass of the cluster M999, in solar masses?

5. **Spherical potentials (2 points)** (2.7 of BT) Astronauts orbiting an unexplored planet find that (i) the surface of the planet is precisely spherical and centered on $r = 0$; and (ii) the potential exterior to the planetary surface is $\Phi = -GM/r$ exactly, that is, there are no non-zero multipole moments other than the monopole. Can they conclude from these observations that the mass distribution in the interior of the planet is spherically symmetric? If not, give a simple example of a non-spherical mass distribution that would reproduce the observations.