Quantum Mechanics 171.605: Problem Set 1

Due: Friday, September 16, 2011 (in conference)

P.1 Which of the following operators are Hermitian and which aren't?

(a) $\partial/\partial x$

(b) $x \cdot \hat{p}$

(c) $3 + 4i$

P.2 If the potential energy function is real, show that the Hamiltonian operator,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

is Hermitian.

P.3 Consider the *angular momentum operator,* defined by

$$\hat{L}_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$

which acts on angular wavefunctions of the form $Z(\phi)$. To be consistently defined, the wavefunctions should satisfy $Z(\phi + 2\pi) = Z(\phi)$. Explain why this is so, and then use this fact to show that the angular momentum operator is Hermitian.

P.4 **(a)** Show that the Schrödinger equation in momentum space can be written in terms of an integral equation involving a "nonlocal" potential energy, namely,

$$\left(E - \frac{p^2}{2m}\right)\phi(p) = \int_{-\infty}^{+\infty} V(p - \overline{p})\phi(\overline{p})d\overline{p}$$

where

$$\overline{V}(q) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} e^{-iqx/\hbar} V(x)\, dx$$

is essentially the Fourier transform of $V(x)$.

(b) Find the form of $\overline{V}(p - \overline{p})$ in the case of the uniformly accelerated particle.

P.5 **(a)** Confirm that $[\hat{x}, p] = i\hbar$ in momentum space.

(b) Show that $[x, \hat{p}^2] = 2i\hbar\hat{p}$ by acting with both sides on an arbitrary function of x.