Quantum Mechanics 171.605: Problem Set 2

Due: Tuesday, September 27, 2011 (in class)

P1. **Well consisting of two delta functions**

Consider a particle of mass m whose potential energy is

$$V(x) = -\alpha\delta(x) - \alpha\delta(x-l) \qquad \alpha > 0$$

where l is a constant length.

a) Calculate the bound states of the particle, setting $E = -\dfrac{\hbar^2\rho^2}{2m}$. Show that the possible energies are given by the relation

$$e^{-\rho l} = \pm\left(1 - \frac{2\rho}{\mu}\right)$$

where μ is defined by $\mu = \dfrac{2m\alpha}{\hbar^2}$. Give a graphic solution of this equation.

(*i*) *Ground state*. Show that this state is even (invariant with respect to reflection about the point $x = l/2$). Find its energy E_S (or at least make an estimate).

(*ii*) *Excited state*. Show that, when l is greater than a value which you are to specify, there exists an odd excited state, of energy E_A greater than E_S. Find the corresponding wave function.

(*iii*) Explain how the preceding calculations enable us to construct a model which represents an ionized diatomic molecule (H_2^+, for example) whose nuclei are separated by a distance l. How do the energies of the two levels vary with respect to l? What happens at the limit where $l \longrightarrow 0$ and at the limit where $l \longrightarrow \infty$? If the repulsion of the two nuclei is taken into account, what is the total energy of the system? Show that the curve which gives the variation with respect to l of the energies thus obtained enables us to predict in certain cases the existence of bound states of H_2^+, and to determine the value of l at equilibrium. In this way we obtain a very elementary model of the chemical bond.

b) Calculate the reflection and transmission coefficients of the system of two delta function barriers. Study their variations with respect to l. Do the resonances thus obtained occur when l is an integral multiple of the de Broglie wavelength of the particle? Why?

P2. Consider a particle placed in the potential

$$V(x) = 0 \qquad \text{if } x \geqslant a$$
$$V(x) = -V_0 \qquad \text{if } 0 \leqslant x < a,$$

with $V(x)$ infinite for negative x. Let $\varphi(x)$ be a wave function associated with a stationary state of the particle. Show that $\varphi(x)$ can be extended to give an odd wave function which corresponds to a stationary state for a square well of width $2a$ and depth V_0. Discuss, with respect to a and V_0, the number of bound states of the particle. Is there always at least one such state, as for the symmetric square well?

P3. Consider, in a two-dimensional problem, the oblique reflection of a particle from a potential step defined by:

$$V(x, y) = 0 \qquad \text{if } x < 0$$
$$V(x, y) = V_0 \qquad \text{if } x > 0$$

Study the motion of the center of the wave packet. In the case of total reflection, interpret physically the differences between the trajectory of this center and the classical trajectory (lateral shift upon reflection). Show that, when $V_0 \longrightarrow +\infty$, the quantum trajectory becomes asymptotic to the classical trajectory.