Quantum Mechanics 171.605: Problem Set 5

Due: Friday, October 28, 2011 (in conference)

P1. Apply the WKB quantization condition to the "half harmonic oscillator" potential, namely,

$$V(x) = \begin{cases} +\infty & \text{for } x < 0 \\ m\omega^2 x^2/2 & \text{for } x > 0 \end{cases}$$

How does the WKB quantization method "pick out" only the odd states of the full potential?

P2. Apply the WKB quantization condition to estimate the bound state energies of the potential

$$V(x) = -\frac{V_0}{\cosh(x/a)^2}$$

(a) Show that your results can be written in the form

$$E_n = -\left(\sqrt{V_0} - (n+1/2)\sqrt{\frac{\hbar^2}{2ma^2}}\right)^2$$

Hint: You might use the integral

$$\int_0^A \frac{\sqrt{A^2-u^2}}{1+u^2}\,du = \frac{\pi}{2}\left(\sqrt{1+A^2}-1\right)$$

(b) If $V_0 \gg \hbar^2/2ma^2$, show that your result approximates the harmonic oscillator approximation for this potential.

(c) One might think that one could take the limit $V_0 \to \infty$ and $a \to 0$ in such a way as to reproduce an attractive δ function potential. Discuss the WKB approximation to the energy levels in this limit. If it works, does it reproduce the known result? If it doesn't, why not?