

Landau Levels and Quasiparticle Spectrum of Extreme Type-II Superconductors

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At high magnetic fields and low temperatures, numerous extreme type-II superconductors exhibit Landau quantization of electronic motion. We present an analytic construction of the quasiparticle spectrum in this regime, based on the high-field expansion. The spectrum is gapless and is separated from the familiar low-field regime by a quantum level-crossing transition. Such low energy excitations should lead to observable effects in thermodynamics and transport. [S0031-9007(97)05116-8]

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Recent observations of the de Haas–van Alphen (dHvA) oscillations in many extreme type-II superconducting materials [1] clearly indicate the presence of Landau level (LL) quantization within a high-magnetic field (H), low-temperature (T) “pocket,” surrounding the $H_{c2}(0)$ point in the H - T phase diagram (Fig. 1). The size of this pocket in conventional type-II systems (like Nb) is expected to be negligible. This is so because, within the BCS theory, the scale of the cyclotron splitting between LLs near $H_{c2}(0)$, $\omega_c \sim \omega_{c2}(0) \equiv eH_{c2}(0)/m^*c$, is set by the condensation energy $\sim T_{c0}^2/E_F$, and should be much smaller than either the thermal smearing $\sim T$ or the gap $\Delta(T, H)$, the scale for both being T_{c0} . Additional smearing due to disorder Γ makes this tiny pocket effectively irrelevant. In clean, intrinsically extreme type-II systems, the situation is significantly different. We define the extreme type-II systems as those with the slope of the upper critical field at T_{c0} , $H'_{c2} \geq 0.3$ T/K [3]. In such systems $\omega_{c2}(0)$ is comparable to T_{c0} and there is a large pocket in the phase diagram in which the LL structure within the superconducting phase is well defined; i.e., $\omega_c > \Delta(T, H), T, \Gamma$. Numerous superconductors belong to this extreme type-II family: high- T_c cuprates, A15's, boro-carbides, many organics, etc. The boundaries of the “extreme” pocket, H^* and T^* , defined by $\omega_c \sim \Delta(T=0, H)$ and $\omega_c \sim T$, extend to H as low as $H^* \sim 50\%H_{c2}(0)$ and T as high as $T^* \sim 30\%T_{c0}$ (Fig. 1) [1].

Within this extreme pocket the solution to the superconducting problem must fully incorporate the LL structure of the normal state. This leads to a set of Bogoliubov–de Gennes (BdG) equations for the LL quantized quasiparticles in the presence of the gap function $\Delta(\mathbf{r})$, describing the Abrikosov vortex lattice [2,4]. There are two main sources of difficulty in solving these equations. First, the number of LLs involved in the pairing is typically 40–100 and the matrix elements of the gap function between these states, $\Delta_{nm}(\mathbf{q})$, are in general quite complex. A more serious difficulty, however, is that the basis which diagonalizes the BCS Hamiltonian involves combined “rotations” both in the Nambu space and the LL basis. To illustrate why this fact seriously impedes analytic progress we first note that, to the leading order in $\Delta/\omega_c \ll 1$, the

quasiparticle excitation spectrum near the Fermi surface (FS) can be found exactly: $E = \pm\sqrt{\varepsilon_n(k_z)^2 + |\Delta_{nm}(\mathbf{q})|^2}$ [2]. It is tempting to generalize this and conclude that the full solution takes the form $E = \pm\sqrt{\Sigma_{\bar{n}\bar{n}}^2 + |D_{\bar{n}\bar{n}}|^2}$, where $\Sigma_{\bar{n}\bar{n}}(\mathbf{q}, k_z)$ and $D_{\bar{n}\bar{n}}(\mathbf{q}, k_z)$ are the normal and pairing self-energies, respectively. The physical meaning of such a solution would be that we first rotate the LL basis, $\{n\} \rightarrow \{\bar{n}\}$, and then diagonally pair up these new “normal” quasiparticles. This would lead to a simple description, suitable for applications of our standard theoretical machinery. Unfortunately, this conclusion would be incorrect: Beyond the leading order, the full solution cannot be put in the above simple form as the normal and “pairing” self-energies cannot be simultaneously diagonalized [5]. This leads to a loss of physical transparency and makes it difficult to interpret results in familiar terms. Most of the work beyond leading order has been numerical [2,4,6,7].

In this Letter we introduce an analytic approach which allows for a transparent and systematic evaluation of corrections to the leading order results. The basic idea can be illustrated by an example of an interacting Fermi liquid: There various properties are in general very difficult

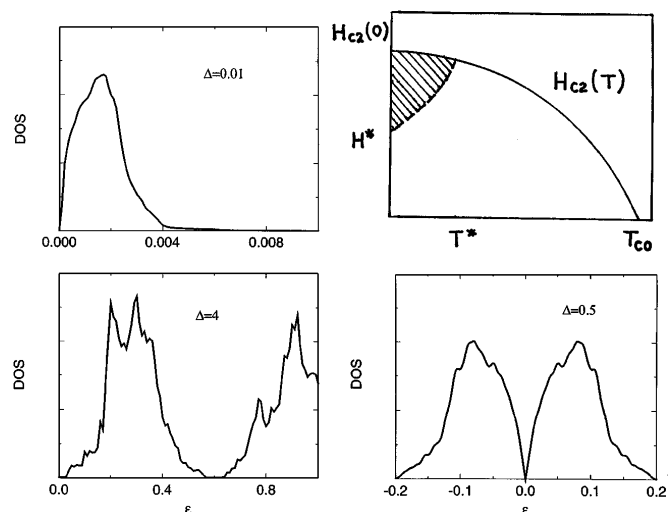


FIG. 1. Density of states (DOS) (in arbitrary units) for several values of Δ (see text). The inset shows the extreme pocket (shaded area).

to evaluate. However, as long as we are interested in the low T thermodynamics and low energy transport, these properties can be accurately described by the *quasiparticle* representation of the excitations near the FS. This leads to the low T thermodynamics which is in one-to-one correspondence with that of noninteracting fermions, but with a modified dispersion relation and a reduced spectral weight. In this paper, we construct such a “quasiparticle” representation of the present problem. Of course, there are no interactions in our case but our aim is similar. The original partition function is replaced by the one of quasiparticles whose energies *do have* the desired form $E = \pm\sqrt{\tilde{\varepsilon}_{nn}^2 + Z^2|\tilde{\Delta}_{nn}|^2}$, and which faithfully represents the low T (or low energy) properties of the original problem. Here $\tilde{\varepsilon}_{nn}(\mathbf{q}, k_z)$ and $\tilde{\Delta}_{nn}(\mathbf{q}, k_z)$ are the *renormalized* normal and pairing self-energies and Z is the quasiparticle renormalization factor. All can be computed to the desired accuracy in Δ/ω_c from a general expression which we provide. Using this quasiparticle representation we derive the following results: First, we evaluate the leading order corrections to normal and pairing self-energies and find that they are of order $\sim\Delta^2/\omega_c$ and $\sim\Delta^3/\omega_c^2$, respectively. Therefore, for $\Delta/\omega_c \ll 1$, the features of the spectrum obtained in the leading order are only moderately affected. In particular, we show that the quasiparticle excitations remain *gapless to all orders in perturbation theory* until Δ/ω_c exceeds a certain critical value. As Δ/ω_c increases further ($H < H^*$), the original LL structure is obliterated by strong LL mixing induced by off-diagonal Δ_{nm} 's. This exact result shows that the *gapless* “extreme” pocket is truly a novel state of type-II superconductors and is separated from the *gapped* low-field mixed phase by a quantum level crossing transition. In this context, we point to an important difference between 3D and 2D systems. We next present our numerical results for the excitation spectrum and find that they are in agreement with the analytic results. We hope to inspire experiments which could test our results and provide novel insight into the physics of high-field, extreme type-II superconductors.

Our starting point is the BdG equations:

$$\begin{aligned} Eu_{k_z, \mathbf{q}, n}^N &= \varepsilon_n(k_z)u_{k_z, \mathbf{q}, n}^N + \sum_m \Delta_{nm}(\mathbf{q})v_{k_z, \mathbf{q}, m}^N, \\ Ev_{k_z, \mathbf{q}, n}^N &= -\varepsilon_n(k_z)v_{k_z, \mathbf{q}, n}^N + \sum_m \Delta_{mn}^*(\mathbf{q})u_{k_z, \mathbf{q}, m}^N, \end{aligned} \quad (1)$$

where $\varepsilon_n(k_z) \equiv \hbar^2 k_z^2/2m + n\hbar\omega_c - \mu$, n is the LL index, $\hbar\omega_c/2$ has been absorbed into the chemical potential μ , \mathbf{q} is the magnetic wave vector, and k_z is the wave vector along the field direction [2]. $\Delta_{nm}(\mathbf{q}) = \sum_j \Delta_{n,m}^j(\mathbf{q})$ is the matrix element of $\Delta(\mathbf{r}) = \sum_j \Delta_j(\mathbf{r})$ [8] between electronic states (k_z, \mathbf{q}, n) and $(-k_z, -\mathbf{q}, m)$ [2]. N is the number of LLs which participate in pairing. The solution of the above equations gives the quasiparticle states and the energy spectrum within the superconducting phase.

As one crosses into the superconducting state and Δ becomes finite, there are both perturbative and nonpertur-

bative effects in Eq. (1). We first isolate the nonperturbative part by noticing that the particle bands (u bands) with the LL index $n_1 = n + p$ and the hole bands (v bands) with the LL index $n_2 = n - p$ ($p = 0, \pm 1, \pm 2, \dots$) cross at energies $p\hbar\omega_c$ whenever $k_z = \pm k_{Fn} \equiv \sqrt{2m(\mu - n\hbar\omega_c)}$, where k_{Fn} are the Fermi wave vectors of different quasi-one-dimensional LL bands. Additional band crossings take place at energies $(p + 1/2)\hbar\omega_c$ and for $k_z = \pm\sqrt{k_{Fn}^2 + m\hbar\omega_c}$. At the band crossings near $k_z = \pm k_{Fn}$ the matrix elements $\Delta_{n+p, n-p}(\mathbf{q})$ must be treated by a degenerate perturbation theory. The same is true for $\Delta_{n+p, n-1-p}(\mathbf{q})$ near $k_z = \pm\sqrt{k_{Fn}^2 + m\hbar\omega_c}$. The effect of all other matrix elements, $\Delta_{n+p, n(-1)-p'}(\mathbf{q})$ ($p \neq p'$), is purely perturbative and can be computed by an expansion in Δ/ω_c . Near H_{c2} , where $\Delta/\omega_c \ll 1$, that effect is small and we obtain the analytic form for the BCS spectrum near $k_z = \pm k_{Fn}$:

$$E_{n,p} = p\hbar\omega_c \pm \sqrt{\varepsilon_n(k_z)^2 + |\Delta_{n+p, n-p}(\mathbf{q})|^2}, \quad (2)$$

where $p = 0, \pm 1, \pm 2, \dots$. A similar expression holds for the spectrum around $k_z \sim \pm\sqrt{k_{Fn}^2 + m\hbar\omega_c}$. Typically, we are interested in the quasiparticle spectrum when we discuss low T (or low energy) properties; i.e., $(T, \omega) \ll \Delta(T, H) < \omega_c$. In that case only the quasiparticles near the FS are important, $k_z \sim \pm k_{Fn}$ and $p = 0$, and it suffices to consider only the $E_{n,p=0} = \pm\sqrt{\varepsilon_n(k_z)^2 + |\Delta_{nn}(\mathbf{q})|^2}$ bands. The excitations from $p \neq 0$ bands (2) are gapped by energies which are too high, $E_{n,p \neq 0}(k_z \sim \pm k_{Fn}) \sim p\hbar\omega_c$, and their contribution to the quasiparticle thermodynamics is negligible at low T [$T \ll \Delta(T, H) \ll \omega_c$]. This is the content of the “diagonal approximation” (DA) for the excitation spectrum [2,9].

The main issue addressed in this paper is how to go beyond Eq. (2) analytically. As one moves further below H_{c2} , Δ/ω_c increases (while still $\ll 1$) and the corrections coming from $\Delta_{n+p, n-p'}(\mathbf{q})$ ($p \neq p'$) must be included. A *pedestrian* perturbation theory, starting from (2), leads to expressions whose physical content is difficult to analyze. The reason is that (2) already involves a full Nambu rotation. Consequently, all other terms generated by the pedestrian perturbation theory are fully Nambu rotated. This makes it impossible to disentangle the normal from the “pairing” part of the problem [5]. In contrast, our standard theoretical machinery works best when we can neatly separate the normal from the pairing part. We propose here a different method of computing the excitation spectrum. Consider the partition function specified by the solution of BdG equations (1), $Z = \prod_{\omega, \mathbf{q}, k_z} Z(i\omega, \mathbf{q}, k_z)$, where

$$Z = \prod_n \int d\psi_n d\bar{\psi}_n \exp \left[\sum_{n,m} \bar{\psi}_n (i\omega I - \mathcal{H})_{nm} \psi_m \right]. \quad (3)$$

Here the set of Grassman variables $\{u_n, v_n\}(i\omega, \mathbf{q}, k_z)$ representing the particle (hole) bands of (1) has been arranged in spinors $\tilde{\psi}_n \equiv (\bar{u}_n \bar{v}_n)$. I and $\mathcal{H}(\mathbf{q}, k_z)$ are two $2N \times 2N$ matrices which are, respectively, a unit matrix and a Hamiltonian whose form is evident from the right hand side of Eq. (1). $\{\omega\}$ is the set of fermionic Matsubara frequencies. Note that $Z(i\omega, \mathbf{q}, k_z)$ is a determinant of a matrix, $Z = \det \|i\omega I - \mathcal{H}\|$. When analytically continued, this is precisely the secular determinant of the BdG equations (1). Thus, the solutions for quasiparticle energies in (1) are the zeros of $Z(i\omega \rightarrow E, \mathbf{q}, k_z)$.

Within our extreme pocket [$T \ll \Delta(T, H) \ll \omega_c$] the main contributions to Z come from $k_z \sim \pm k_{Fn}$ and $E_{n,p=0}$ bands. In order to evaluate $Z(i\omega, \mathbf{q}, k_z \sim k_{Fn})$ we first integrate out all $\psi_{n,p \neq 0}(i\omega, \mathbf{q}, k_z \sim k_{Fn})$ in (3):

$$Z = Z_{p \neq 0} \int d\psi_{n,0} d\bar{\psi}_{n,0} \exp[\bar{\psi}_{n,0}(i\omega I - \tilde{\mathcal{H}})_{00}\psi_{n,0}]. \quad (4)$$

$Z_{p \neq 0}(i\omega, \mathbf{q}, k_z \sim k_{Fn})$ is what one gets from (3) with the $E_{n,p=0}$ band *excluded*. It describes the contribution to thermodynamics arising from all other bands, $E_{n,p \neq 0}$, by themselves. Since $p \neq 0$ bands are far from the FS, $E_{n,p \neq 0}(\mathbf{q}, k_z \sim k_{Fn}) \sim p\hbar\omega_c$, such contributions are activated, i.e., $\sim \exp(-p\hbar\omega_c/T) \ll 1$, and their contribution is very small. Thus, the dominant contribution comes from $\det \|(i\omega I - \tilde{\mathcal{H}})_{00}\|$. Here I_{00} is a 2×2 unit matrix while $\tilde{\mathcal{H}}_{00} \equiv \mathcal{H}^0 + \mathcal{H}'$ is a 2×2 Nambu matrix. The diagonal elements of $\{\mathcal{H}^0, \mathcal{H}'\}(i\omega, \mathbf{q}, k_z \sim k_{Fn})$ are $\{\pm \varepsilon_n(k_z), \pm \Sigma_{nn}(i\omega, \mathbf{q}, k_z)\}$ while the off-diagonal ones are $\{\Delta_{nn}(\mathbf{q}), D_{nn}(i\omega, \mathbf{q}, k_z)\}$ and their complex conjugates (c.c.). By itself, \mathcal{H}^0 determines the spectrum of quasiparticles at the FS (described by $\psi_{n,p=0}$) in the DA (2). \mathcal{H}' is a correction arising from coupling to other spinor bands $\psi_{n,p \neq 0}$, with Σ_{nn} and D_{nn} being the normal and pairing self-energies, respectively. Since the $p \neq 0$ bands are separated from the FS by $p\hbar\omega_c$, these self-energies contain denominators $\sim p\hbar\omega_c$ and are small for $\Delta/\omega_c \ll 1$. The explicit form is obtained from

$$\mathcal{H}' = \sum_{p \neq 0} \sum_{p' \neq 0} \mathcal{H}_{0p}(i\omega I - \mathcal{H})_{pp'}^{-1} \mathcal{H}_{p'0}. \quad (5)$$

\mathcal{H}_{0p} is a 2×2 Nambu matrix whose diagonal elements are zero and off-diagonal ones $\Delta_{n,n+p}(\mathbf{q})$ and its c.c. The resolvent appearing in (5) is that of the BdG system (1) for $k_z \sim k_{Fn}$ but with the $p = 0$ spinor band (2) excluded.

Up to this point there are no approximations and the expression for \mathcal{H}' is formally exact. The problem is that the resolvent for the $p \neq 0$ bands involves inversion of a large matrix. However, for $\Delta \ll \omega_c$, the resolvent can be calculated perturbatively, to a desired accuracy in Δ/ω_c . In this way we obtain the leading order corrections:

$$\begin{aligned} \Sigma_{nn} &= \sum_{p \neq 0} \frac{|\Delta_{n,n+p}(\mathbf{q})|^2}{i\omega + \varepsilon_{n+p}(k_z)}; \\ D_{nn} &= \sum_{p \neq 0, p' \neq 0} \frac{\Delta_{n,n+p}(\mathbf{q}) \Delta_{n+p,n+p'}^*(-\mathbf{q}) \Delta_{n+p',n}(\mathbf{q})}{[i\omega + \varepsilon_{n+p}(k_z)][i\omega - \varepsilon_{n+p'}(k_z)]}. \end{aligned} \quad (6)$$

$\Sigma_{nn}(i\omega, \mathbf{q}, k_z)$ and $D_{nn}(i\omega, \mathbf{q}, k_z)$ are the normal and pairing self-energies, respectively. They have a frequency dependence arising from the coupling to the $p \neq 0$ spinor bands through the matrix elements $\Delta_{n,n+p}(\mathbf{q})$. We can view them as self-energies of some “interacting” system and proceed to construct the quasiparticle representation of (3). This construction uses a *Fermi surface expansion* of (6). First, we determine the *renormalized* quasiparticle energies from $\tilde{\varepsilon}_{nn}(\mathbf{q}, k_z) = \varepsilon_n(k_z) + \Sigma_{nn}(i\omega = \tilde{\varepsilon}_{nn}(\mathbf{q}, k_z), \mathbf{q}, k_z)$. To the leading order in Δ/ω_c

$$\tilde{\varepsilon}_{nn}(\mathbf{q}, k_z) \approx \varepsilon_n(k_z) + \sum_{p \neq 0} \frac{|\Delta_{n,n+p}(\mathbf{q})|^2}{p\omega_c}. \quad (7)$$

We also evaluate the quasiparticle renormalization factor $Z_n^{-1} \equiv 1 - \partial \Sigma_{nn} / \partial i\omega |_{i\omega = \tilde{\varepsilon}_{nn}}$; to the leading order $Z_n \approx 1 - \sum_{p \neq 0} |\Delta_{n,n+p}(\mathbf{q})|^2 / p^2 \omega_c^2$. Similarly, we can compute the “renormalized” shape of the FS. The renormalized Fermi velocity is given by

$$\frac{\tilde{v}_{Fn}(\mathbf{q})}{v_{Fn}} \approx 1 - \sum_{p \neq 0} \frac{|\Delta_{n,n+p}(\mathbf{q})|^2}{p^2 \omega_c^2}. \quad (8)$$

$\tilde{\varepsilon}_{nn}$ and \tilde{v}_{Fn} determine quasiparticle DOS at the FS and their \mathbf{q} dependence describes the “broadening” of LLs by the potential scattering induced through the spatial nonuniformity of $\Delta(\mathbf{r})$. This effect is *secondary* to the leading pairing-induced “splitting” of LLs in Eq. (2). Finally, the renormalized pairing matrix element at the FS

$$\begin{aligned} \tilde{\Delta}_{nn}(\mathbf{q}) &= \Delta_{nn}(\mathbf{q}) - \sum_{p \neq 0, p' \neq 0} \frac{\Delta_{n,n+p}(\mathbf{q}) \Delta_{n+p,n+p'}^*(-\mathbf{q}) \Delta_{n+p',n}(\mathbf{q})}{pp' \omega_c^2}. \end{aligned} \quad (9)$$

This quasiparticle construction is not an ordinary perturbation theory. It keeps the normal and pairing self-energies separate by postponing the Nambu rotation until the last step. It can be viewed as a *quasiparticle representation of the BCS quasiparticle spectrum*.

The above construction can be carried out to any desired degree of accuracy in Δ/ω_c , although the amount of algebra rapidly increases. Certain general statements, however, can be made. In particular, the excitation spectrum determined by the BdG equations (1) is *gapless* in a *finite* interval $\Delta/\omega_c \in [0, x_c]$, where $x_c \neq 0$ is some critical value. This property can be demonstrated by noticing first that all $\Delta_{nm}(\mathbf{q})$ have a set of common zeros whose location in the \mathbf{q} space depends only on whether $n + m$ is even or odd. These are so-called Eilenberger zeros, $\{\mathbf{q}_i^E\}$ [2]. In the leading order (2) the gap is determined by the diagonal matrix elements, $\Delta_{nn}(\mathbf{q})$, and it vanishes at $\mathbf{q} = \mathbf{q}_i^E$ as well as at numerous other points $\{\mathbf{q}_j^n\}$ within the magnetic Brillouin zone (MBZ). As the matrix elements coupling different $\psi_{n,p}$'s are turned on, it is clear from (5) that $D_{nn}(\mathbf{q} = \mathbf{q}_i^E)$ involves only terms with an *odd* number of matrix elements $\Delta_{n+p,n+p'}(\mathbf{q})$, i.e.,

$$D_{nn} \sim \Delta_{n,n+p} \Delta_{n+p,n+p_1} \Delta_{n+p_1,n+p_2} \cdots \Delta_{n+p_k,n}. \quad (10)$$

Note now that it is impossible to form such a string (10) of odd number of Δ 's without at least one $\Delta_{n+p'+n+p''}(\mathbf{q})$ having $n + p' + n + p''$ even. But such a matrix element also must vanish at the same $\{\mathbf{q}_i^E\}$ as $\Delta_{nn}(\mathbf{q})$. Consequently, at these points, $\mathbf{q} = \mathbf{q}_i^E$, the spectrum remains gapless. This statement is correct to all orders in perturbation theory and therefore is exact as long as perturbative expansion itself is well defined. We expect that the radius of convergence of the perturbative expansion extends from $\Delta/\omega_c = 0$ to $\Delta/\omega_c = x_c \sim 1$ [2,10]. At this critical point, the perturbation theory breaks down due to band crossings between neighboring LL branches. For $\Delta/\omega_c > x_c$ the gaps open up at $\{\mathbf{q}_i^E\}$ signaling the destruction of the LL structure by strong mixing due to increasing Δ . Ultimately, for $\Delta/\omega_c \gg x_c$, there is a crossover to the low- H regime of minigapped states in well-separated vortex cores [2], as established by Norman *et al.* [4]. This phenomenon of band crossings and disappearance of gapless excitations has already been noticed in numerical solutions of the BdG equations [2]. Here we elucidate its physical origin through analytic arguments.

We now compare our analytic results with the numerical solution of the BdG equations (1). Such a solution is very time consuming in the 3D case so we have solved (1) for the 2D case. Our analytic results can be adapted to the 2D case but with one key difference: In a 2D system the FS is a *line*, not a *sheet*. Consequently, the gapless and "near" gapless points in the MBZ might not be located on the FS. In contrast, the Fermi "sheet" of a LL quantized 3D material contains *all* of the MBZ. As a result, the gapless character of the excitation spectrum will be less pronounced in 2D systems even though the effects of LL quantization itself are more in evidence there than in quasi-3D materials. This difference between 3D and 2D systems should lead to observable effects in dHvA oscillations [2,4,11], thermodynamics, and transport.

We obtain the spectrum as a function of Δ for different $n_c \equiv \mu/\hbar\omega_c$. The eigenvalues are grouped in pairs centered around the LL energies. As Δ grows, the bands start to broaden until eventually the gap between them goes to zero and the band crossing occurs. We fixed μ such that $\{\mathbf{q}_i^E\}$ points remain gapless. In 2D, if μ is adjusted so that an originally gapless point in the MBZ (2) is on the FS, then the gap opened by the off-diagonal terms should be due to the pairing term and thus $\sim \Delta^3$. In general, however, the normal contributions at $\mathbf{q} = \{\mathbf{q}_j^{nn}\}$ gapless points are finite and their gaps will be $\sim \Delta^2$. A fit of the normal and pairing self-energy contributions for $\mathbf{q} = \mathbf{q}_j^{nn}$ as a function of μ shows that there is a value of μ for which the coefficient of the Δ^2 term vanishes and therefore close to this point the Δ^3 term prevails. This provides a criterion for locating the "renormalized" 3D FS at all $\{\mathbf{q}_i^E, \mathbf{q}_j^{nn}\}$ [10].

The shape of the density of states at low energies is determined by the first and second order zeros. The exact DOS for small Δ and DOS within the DA are basically equal since $\Delta/\omega_c \ll 1$. There is therefore a large DOS at low energies. As Δ grows a gap appears in 2D. Yet, if μ is

set to emulate 3D, the DOS remains similar to the DA. As n_c grows, the region over which the DOS spreads becomes a smaller fraction of Δ , since the number of zeros grows with n_c and the effect of the off-diagonal perturbations decreases, likely due to large cancellations in (7). In Fig. 1 we show the DOS for $n_c = 40$ and $N = 10$ [2,4] for the values of $\Delta = 0.01, 0.5, 4$. The energy is rescaled by ω_c . The nearly diagonal case, $\Delta = 0.01$, and $\Delta = 0.5$ appear very similar. They both show a linear dependence in the DOS indicating qualitatively the same behavior. The case $\Delta = 4$ is qualitatively different. A gap is appearing at low energies and the next band has a small gap to the lowest one indicating that band crossing is imminent. These results demonstrate that within our pocket ($H > H^*, T < T^*$) the DOS is qualitatively similar to the DA and gapless and nearly gapless points $\{\mathbf{q}_i^E, \mathbf{q}_j^{nn}\}$ still dominate low energy properties.

Our results have observable consequences for a broad range of thermodynamic and dynamic properties. Specific heat, ultrasound attenuation, STM and other forms of tunneling, and many other phenomena will exhibit power-law-like behaviors, characteristic of low-energy BCS quasiparticles. All these diagnostic tools can be used to explore the extreme pocket, identify gapless behavior, and test our prediction that the high- and the low-field regime of type-II superconductors differ in this essential respect.

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