

Quantum Criticality of d -Wave Quasiparticles and Superconducting Phase Fluctuations

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We present finite temperature (T) extension of the $(2 + 1)$ -dimensional QED (QED_3) theory of underdoped cuprates. The theory describes nodal quasiparticles whose interactions with quantum proliferated $hc/2e$ vortex-antivortex pairs are represented by an emergent $U(1)$ gauge field. Finite T introduces a scale beyond which the spatial fluctuations of vorticity are suppressed. As a result, the spin susceptibility of the pseudogap state is bounded by T^2 at low T and crosses over to $\sim T$ at higher T , while the low- T specific heat scales as T^2 , reflecting the thermodynamics of QED_3 . The Wilson ratio vanishes as $T \rightarrow 0$; the pseudogap state is a “thermal (semi)metal” but a “spin-charge dielectric.” This non-Fermi liquid behavior originates from two general principles: spin correlations induced by “gauge” interactions of quasiparticles and fluctuating vortices and the “relativistic” scaling of the $T = 0$ fixed point.

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Recent experiments [1–3] provide support for the view that the pseudogap state of copper oxides represents a phase-disordered superconductor [4]. This view is central to the $(2 + 1)$ -dimensional QED (QED_3) theory of underdoped cuprates [5] whose degrees of freedom, Bogoliubov–de Gennes (BdG) quasiparticles and fluctuating $hc/2e$ vortex-antivortex pairs, and their mutual interactions are argued to capture the effective low-energy physics of a d -wave superconductor in a doped Mott insulator. Within the pseudogap phase of high- T_c cuprates, this QED_3 theory assumes the role played by the Fermi liquid theory in conventional metals and superconductors. The theory possesses three major dynamical symmetries: relativistic, gauge invariance, and chiral symmetry, all three emergent in nature [6]. Irrespective of whether the symmetric phase of QED_3 is the true $T = 0$ ground state of the system, these symmetries conspire to make its peculiar brand of quantum criticality a strongly attractive basin of influence on physical properties of the pseudogap state.

In this Letter, we focus on the subset of such properties which are intrinsically important features of a condensed matter system: thermodynamics and spin response of the pseudogap state. Our primary aim is to stimulate experimental activity by providing explicit and testable theoretical predictions stemming from the theory of Ref. [5]. The summary of our results is as follows: First, in order to derive thermodynamics and spin susceptibility from QED_3 theory, we generalize its form to finite T . This is a matter of some subtlety since, the moment $T \neq 0$, the theory loses its fictitious “relativistic invariance.” Second, we show that the theory predicts a finite T scaling form for thermodynamic quantities in the pseudogap state and propose that this form be tested experimentally. Third, we determine the leading $T \neq 0$ scaling and demonstrate that the deviations from “relativistic invariance” are actually irrelevant for T much less than the pseudogap temperature T^* , in the sense that the leading order $T \neq 0$

scaling of thermodynamic functions remains that of the finite- T symmetric QED_3 [7]. These deviations from relativistic invariance do, however, affect higher order terms. Finally, we evaluate the uniform magnetic spin susceptibility χ and show that it is bounded by T^2 at low T but crosses over to $\sim T$ at higher T , closer to T^* . Consequently, the Wilson ratio $R = \chi T / c_v$ vanishes as $T \rightarrow 0$: the QED_3 theory implies the non-Fermi liquid nature of the pseudogap state in cuprates. Such a state is a thermal (semi)metal but a spin and charge dielectric [8]. Our results are thus suggestive of the breakdown of Wiedemann-Franz law in the pseudogap state.

The spin susceptibility of a d -wave superconductor vanishes linearly with temperature. The way to understand this result is to notice that the spin part of the ground state wave function, being a spin singlet, remains unperturbed by the application of a weak uniform magnetic field. However, the excited quasiparticle states are not, in general, spin singlets and therefore contribute to the finite temperature susceptibility. Because their density of states is linear at low energies, at finite temperature the number of quasiparticles that are excited is $\sim k_B T$, each contributing a constant to the Pauli-like uniform spin susceptibility χ . Thus $\chi \sim T$.

When the superconducting phase order is destroyed by proliferation of unbound quantum vortex-antivortex pairs, the low-energy quasiparticles are strongly interacting. The interaction originates from the fact that it is the spin singlet pairs that acquire one unit of angular momentum in their center of the mass coordinate, carried by an $hc/2e$ vortex. This translates into topological frustration in the propagation of BdG “spinon” excitations. As a result, nontrivial spin correlations persist in the excited states of the phase-disordered d -wave superconductor. At low temperatures, these correlations can be described by an emergent $U(1)$ gauge field [5] and lead to suppression of χ relative to its value for noninteracting BdG quasiparticles. We shall argue below that $\chi \lesssim T^2$.

Similarly, in a d -wave superconductor, linear density of the quasiparticle states translates into a T^2 dependence of the low- T specific heat. When the interactions between quasiparticles are included, the spectral weight is transferred to multiparticle states. Within QED₃ theory, however, the strongly interacting infrared (IR) fixed point possesses emergent “relativistic invariance” and the dynamical critical exponent $z = 1$. Furthermore, the effective quantum action for vortices, deep in the phase-disordered pseudogap state, introduces an additional length scale, the superconducting correlation length $\xi_{\tau,\perp}$ (labels τ and \perp stand for timelike and spacelike, respectively). At $T = 0$, this scale serves as a short distance cutoff of the theory and is generically doping (x) dependent. We then argue that under rather general circumstances the low- T ($T \ll T^*$) electronic specific heat scales as T^2 while the free energy goes as T^3 . We now proceed to substantiate the above claims.

Vorticity fluctuations.—Deep in the phase-disordered pseudogap state, the fluctuations in the vorticity three-vector $b_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda$ are described by the following “bare” Lagrangian:

$$\begin{aligned} \mathcal{L}_0[a_\mu] = & \frac{K}{2} f_\perp \left(\frac{T}{\mathbf{k}}, \frac{T}{\omega_n}, KT, K_\tau T \right) b_0^2 \\ & + \frac{K_\tau}{2} f_\tau \left(\frac{T}{\mathbf{k}}, \frac{T}{\omega_n}, KT, K_\tau T \right) \mathbf{b}^2, \end{aligned} \quad (1)$$

where ω_n is Matsubara frequency, K, K_τ are related to the finite superconducting correlation length of the pseudogap state as $K_\tau \propto \xi_{sc}^2 / \xi_\tau$, $K \propto \xi_\tau$ [5], and we have also set $v_F = v_\Delta = 1$ for simplicity. $f_{\tau,\perp}$ are general scaling functions describing how $\mathcal{L}_0[a_\mu]$ is modified from its relativistically invariant $T = 0$ form as the temperature is turned on. They involve only the thermal length $\sim v_F/T$ and K, K_τ and satisfy the condition $f_\tau(0, 0, 0, 0) = f_\perp(0, 0, 0, 0) = 1$. Physically, the modifications embodied in $f_{\tau,\perp}$ are due to changes in the pattern of vortex-antivortex fluctuations induced by finite T . The explicit expressions for $f_{\tau,\perp}$ depend on the details of a particular model for phase disorder within the pseudogap state—as we emphasize below, however, our results are either completely insensitive to such details or reflect only the most general features of $f_{\tau,\perp}$.

To handle the intrinsic space-time anisotropy, it is convenient to introduce two tensors:

$$\begin{aligned} A_{\mu\nu} = & \left(\delta_{\mu 0} - \frac{k_\mu \omega_n}{k^2} \right) \frac{k^2}{\mathbf{k}^2} \left(\delta_{0\nu} - \frac{\omega_n k_\nu}{k^2} \right), \\ B_{\mu\nu} = & \delta_{\mu i} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \delta_{j\nu}, \end{aligned} \quad (2)$$

and rewrite the gauge field action as

$$\mathcal{L}_0[a_\mu] = \frac{1}{2} \Pi_A^0 a_\mu A_{\mu\nu} a_\nu + \frac{1}{2} \Pi_B^0 a_\mu B_{\mu\nu} a_\nu. \quad (3)$$

In the above equations $k_\mu = (\omega_n, \mathbf{k})$, i.e., $k^2 = \omega_n^2 + \mathbf{k}^2$. It is straightforward to show that

$$\begin{aligned} \Pi_A^0 = & K_\tau f_\tau \left(\frac{T}{\mathbf{k}}, \frac{T}{\omega_n}, KT, K_\tau T \right) (\mathbf{k}^2 + \omega_n^2), \\ \Pi_B^0 = & K_\tau f_\tau \left(\frac{T}{\mathbf{k}}, \frac{T}{\omega_n}, KT, K_\tau T \right) \omega_n^2 \\ & + K f_\perp \left(\frac{T}{\mathbf{k}}, \frac{T}{\omega_n}, KT, K_\tau T \right) \mathbf{k}^2. \end{aligned} \quad (4)$$

The gauge field a_μ couples minimally to N Dirac spinors representing nodal BCS quasiparticles [5] ($N = 2$ for a single CuO₂ layer). Consequently, the resulting Lagrangian reads

$$\mathcal{L} = \bar{\psi} (i\gamma_\mu \partial_\mu + \gamma_\mu a_\mu) \psi + \mathcal{L}_0[a_\mu], \quad (5)$$

where the summation over N fermion flavors is understood. The integration over Berry gauge field a_μ reproduces the interaction among quasiparticles arising from the topological frustration referred to earlier.

Specific heat and QED₃ scaling of thermodynamics.—The only length scales that appear in the thermodynamics are the thermal length $\sim v_F/T$, and the superconducting correlation lengths K, K_τ . At $T = 0$, the two correlation lengths K, K_τ enter only as effective short distance cutoffs of the theory since the electronic action is controlled by the IR fixed point of QED₃. These observations allow us to write down the general scaling form for the doping-dependent free energy of the pseudogap state,

$$\mathcal{F}(T; x) = -\frac{T^3}{v_F^2} \Phi_N(K_\tau(x)T, K(x)T) + \mathcal{E}_0(x), \quad (6)$$

where $\Phi_N(x, y)$ is the thermodynamic scaling function and $\mathcal{E}_0(x)$ is the reference ground state energy. Note that we still maintain $v_F = v_\Delta$ for simplicity and will discuss Dirac cone anisotropy shortly. In the above scaling form K_τ is directly related to the $T \rightarrow 0$ finite superconducting correlation length of the pseudogap state, $\xi_{sc}(x)$ [5]. The ratio K_τ/K describes the anisotropy between timelike and spacelike vortex fluctuations and is also a function of doping x . The scaling expressions for other thermodynamic functions can be derived from (6) by taking appropriate temperature derivatives.

We are interested in the low temperature regime ($T \ll T^*$) in which the thermal length v_F/T is much longer than K_τ and K [$v_F/T \gg \xi_{sc}(x)$] or, equivalently, in the limit $\Phi_N(x \rightarrow 0, y \rightarrow 0)$. This is just the limit in which the free energy \mathcal{F} (6) approaches the free energy of the finite temperature QED₃ and the precise form of (1) becomes unimportant as long as $f_{\tau,\perp}$ remain finite as $T \rightarrow 0$. Consequently, $\Phi_N(x, y)$ should be regular at $x = y = 0$ and takes on the value C_{QED} which is a universal positive numerical constant within QED₃ [9,10] (see Refs. [11,12] for a discussion of related issues). Therefore, we finally find that, at $T \ll T^*$, $\mathcal{F} = -C_{\text{QED}} T^3 / v_F^2$, $S = 3C_{\text{QED}} T^2 / v_F^2$, and $c_v = 6C_{\text{QED}} T^2 / v_F^2$.

The physical origin of the above form for the low temperature scaling can be illustrated by the expression for entropy consisting of the free fermion contribution and the part generated solely through gauge field (vortex fluctuation) mediated interactions [10]:

$$S = NS_f^0 + S_{\text{int}} = NC_f T^2 - \frac{\partial}{\partial T} \Delta F, \quad (7)$$

where, for $N = 2$, $C_f = 9\zeta(3)/8\pi v_F v_\Delta$, and

$$\Delta F = \sum_{X=A,B} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \coth\left(\frac{\omega}{2T}\right) \times \tan^{-1}\left(\frac{\text{Im} \Pi_X^{F,\text{ret}}}{\text{Re} \Pi_X^{F,\text{ret}} + \text{Re} \Pi_X^{0,\text{ret}}}\right). \quad (8)$$

Note that we have traded the sum over Matsubara frequencies $\{\omega_n\}$ for an integral over real frequency ω [13].

The retarded fermion polarization function $\Pi_X^{F,\text{ret}}$ can be written as $NT\mathcal{P}_X^F(|\mathbf{k}|/T, \omega/T)$ while the bare gauge field polarization $\Pi_X^{0,\text{ret}}$ has the form $KT^2\mathcal{P}_X^0(|\mathbf{k}|/T, \omega/T, KT, K_\tau T)$. By rescaling momenta and frequencies appearing in Eq. (8), we see that, in the low temperature limit $KT \ll 1$ [$T \ll T^*$ or $v_F/T \gg \xi_{\text{sc}}(x)$], the bare gauge field polarization alters only the higher order corrections to the leading $S \sim T^2$ scaling of QED₃ which itself is determined exclusively by the fermion polarization. Equivalently, the finite temperature modifications (1) to the original QED₃ theory of nodal quasiparticles interacting through vortex-antivortex fluctuations are in effect corrections to the UV cutoff of the pure 2D quantum electrodynamics. However, the leading order scaling of the thermodynamics of pure QED₃ is dominated entirely by the IR fixed point, and is consequently independent of the UV cutoff.

Spin susceptibility.—The “topological” fermion spinors ψ [5] allow us to express the physical spin density $\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2$ as a Dirac fermion density $\bar{\psi} \gamma_0 \psi$. Consequently, the theory connects correlations among topologically frustrated BdG spinons to charge fluctuations in QED₃. Such correlations suppress spin response in phase-disordered underdoped cuprates and, at $T = 0$, one finds $\chi \sim q^2$. Of course, within pure QED₃ the charge fluctuations would still vanish even at finite T since that system is always incompressible. Within the QED₃ theory of cuprates, however, we must now use the “nonrelativistic” modified finite T bare Lagrangian (1). As shown below, this translates into a finite spin susceptibility whose precise value reflects the static limit of the scaling function f_τ . This is in contrast to the thermodynamics where the leading low temperature behavior was determined by the pure relativistically invariant QED₃ and is insensitive to the specific structure of $f_{\tau,\perp}$.

To compute the spin-spin correlation function $\langle S_z(-k)S_z(k) \rangle$, we introduce an auxiliary source $J_\mu(k)$ and couple it to fermion three current. Thus,

$$\mathcal{L}[\bar{\psi}, \psi, a_\mu, J_\mu] = \bar{\psi}[i\gamma_\mu \partial_\mu + \gamma_\mu(a_\mu + J_\mu)]\psi + \mathcal{L}_0[a_\mu], \quad (9)$$

and, since it is the z component of the spin that couples to the gauge field, we have

$$\langle S_z(-k)S_z(k) \rangle = \frac{1}{Z[J_\mu]} \frac{\delta}{\delta J_0(-k)} \frac{\delta}{\delta J_0(k)} Z[J_\mu] \Big|_{J_\mu=0}, \quad (10)$$

Z being the quantum partition function. Now we set $a'_\mu = a_\mu + J_\mu$ and integrate out both the fermions and the gauge field a'_μ . The correlations between a'_μ fields are described by the polarization matrix which to the order $1/N$ can be written in the form $\Pi_{\mu\nu} = (\Pi_A^0 + \Pi_A^F)A_{\mu\nu} + (\Pi_B^0 + \Pi_B^F)B_{\mu\nu}$. The resulting spin correlation function is then readily found to be

$$\langle S_z(-k)S_z(k) \rangle = \frac{\Pi_A^F \Pi_A^0}{\Pi_A^F + \Pi_A^0} \frac{\mathbf{k}^2}{\mathbf{k}^2 + \omega_n^2}, \quad (11)$$

where Π_A^F denotes the fermion current polarization function. Because of the scale invariance of the (massless) topological fermion action, the time component of the retarded polarization function has the scaling form

$$\Pi_A^{F,\text{ret}}(|\mathbf{k}|, \omega, T) = NT P_A^F\left(\frac{|\mathbf{k}|}{T}, \frac{\omega}{T}\right), \quad (12)$$

where $P_A^F(x, y)$ is a universal function of its arguments and N is the number of the four component Dirac fermion species ($N = 2$ for a single CuO₂ layer). In the static limit, $\omega \rightarrow 0$,

$$\lim_{y \rightarrow 0} \text{Re} P_A^F(x, y) = \frac{2 \ln 2}{\pi} + \frac{x^2}{24\pi} + \mathcal{O}(x^3); \quad x \ll 1, \quad (13)$$

and

$$\lim_{y \rightarrow 0} \text{Re} P_A^F(x, y) = \frac{x}{8} + \frac{6\zeta(3)}{\pi x^2} + \mathcal{O}(x^{-3}); \quad x \gg 1, \quad (14)$$

while

$$\lim_{y \rightarrow 0} \text{Im} P_A^F(x, y) = \frac{2 \ln 2}{\pi} \frac{y}{x} + \mathcal{O}\left(\frac{y^2}{x^2}\right); \quad x \ll 1. \quad (15)$$

Furthermore, in order to complete the computation of spin susceptibility, we observe that in the limit $\omega = 0$ and $\mathbf{k} \rightarrow 0$ we have

$$f_\tau\left(\frac{T}{|\mathbf{k}|}, \infty, KT, K_\tau T\right) \rightarrow 1 + c \frac{T^2}{\mathbf{k}^2}, \quad (16)$$

where c is a pure number [14]. The expression (16) for the static limit of f_τ is rather general and can be understood on physical grounds: In the pseudogap state at $T \rightarrow 0$ we can think of the “normal” system as a dual superfluid of relativistic “vortex bosons” [5]. These bosons are coupled to a massless dual gauge field which describes the original

long range vortex-vortex interactions. Once T is finite, this dual superfluid immediately loses its off-diagonal order since the screening by the dual gauge field allows for creation of a finite density of thermally activated dual vortices, i.e., “vortices” in the dual superfluid of original vortices. Thus, there is no finite T transition in the dual superfluid that would be analogous to the Kosterlitz-Thouless transition in original superconductor. The above form for f_τ (16) is just the mathematical expression of this fact.

Combining all of these results together, we finally obtain the spin susceptibility:

$$\chi(\omega = 0, |\mathbf{k}| \rightarrow 0) = \frac{2N \ln 2}{\pi} \frac{K_\tau T^2}{\frac{2N \ln 2}{\pi c} + K_\tau T}. \quad (17)$$

Consequently, Wilson ratio $R = \chi T / c_v$ vanishes as $T \rightarrow 0$ [14] implying the non-Fermi liquid nature of the pseudogap state in cuprates within the theory of Ref. [5].

Corrections to scaling.—There are several sources of corrections to the QED₃ scaling, and it is not practical to address them all within the confines of this Letter. Instead, we choose to concentrate here on those particular corrections to scaling which arise from the Dirac cone anisotropy. Such corrections are most likely to be important in real materials where the values of velocities v_F and v_Δ as functions of underdoping are now becoming known with increasing precision [15]. It was shown in Ref. [16] that the Dirac cone anisotropy $\alpha_D = v_F / v_\Delta$ scales to unity at the QED₃ IR fixed point; alternatively, when defined via $\alpha_D \equiv 1 + \delta$, δ has a small, but finite scaling dimension $\eta_\delta > 0$. To the leading order in $1/N$, $\eta_\delta = 32 / (5\pi^2 N)$ [16]. For simplicity, we assume here that the bare action for the gauge field is $e^{-2} F_{\mu\nu}^2$ ($K = e^{-2}$), where $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. Then the T -dependent part of the free energy scales as

$$\mathcal{F} = -\frac{T^3}{v_F^2} \Phi_N\left(\frac{T}{e^2}, \frac{v_F}{v_\Delta}\right) = -\frac{T^3}{v_F^2} \Phi_N\left(\frac{T}{e^2}, 1 + \delta(T)\right), \quad (18)$$

and in the limit of $T \rightarrow 0$ we have

$$\mathcal{F} = -\frac{T^3}{v_F^2} \Phi_N(0, 1) - \frac{T^{3+\eta_\delta}}{v_F^2} \delta_0 \Phi'_N(0, 1), \quad (19)$$

where Φ_N and Φ'_N are pure numbers and $\delta_0 = \alpha_D - 1$. Thus, although the leading order scaling in T is analytic, just as in the pure finite- T QED₃, the fact that the next-to-leading order correction is not reflects the strongly interacting nature of the QED₃ IR fixed point.

In summary, we have shown that the leading order low temperature scaling of the thermodynamic functions in the pseudogap state is that of a pure finite- T QED₃, since the finite temperature modifications (1) to the theory of Ref. [5] affect only corrections to scaling. As a result, the specific heat goes as $c_v \sim T^2$. Furthermore, we argued that at low temperatures the spin susceptibility is sup-

pressed, $\chi \lesssim T^2$, due to correlations among BdG spinons mediated by an emergent gauge field a_μ . This implies a vanishing Wilson ratio and breakdown of the Fermi liquid behavior within the pseudogap state. Finally, the corrections to scaling arising from Dirac cone anisotropy were argued to be nonanalytic, representing yet another manifestation of the strongly interacting character of the QED₃ brand of quantum criticality.

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