Charge Modulation, Spin Response, and Dual Hofstadter Butterfly in High-T_c Cuprates

Zlatko Tešanović
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218, USA
(Received 28 April 2004; published 17 November 2004)

The modulated density of states observed in recent STM experiments in underdoped cuprates is argued to be a manifestation of the charge-density wave of Cooper pairs (CPCDW). CPCDW formation is due to superconducting phase fluctuations enhanced by Mott-Hubbard correlations near half-filling. The physics behind the CPCDW is related to a Hofstadter problem in a dual superconductor. It is shown that CPCDW does not impact nodal fermions at the leading order. An experiment is proposed to probe coupling of the CPCDW to the spin carried by nodal quasiparticles.

DOI: 10.1103/PhysRevLett.93.217004 PACS numbers: 74.20.Mn, 74.40.+k

Recent STM experiments [1–3] have reinvigorated the debate [4,5] on the nature of the pseudogap state in underdoped cuprates. The central issue is whether the pseudogap state is a phase disordered superconductor [6–9] or some other, an entirely different form of competing order, originating from the particle-hole channel [5,10–12]. The observed modulation in the local density of states (DOS), which breaks the lattice translational symmetry of CuO_2 planes, is conceivably attributable to both.

Within the phase-fluctuating superconductor scenario, a natural temptation is to ascribe this modulation to "helium physics": a system of bosons (Cooper pairs) with short range repulsion is superfluid in its ground state as long as it is *compressible* [13]—the only alternative to superfluidity is an *incompressible* state [14]. In cuprates, as doping x is reduced toward half-filling, $x \to 0$, strong on-site repulsion suppresses particle density fluctuations and reduces compressibility. This leads to enhanced phase fluctuations and a reduced superfluid density ρ_s, courtesy of the uncertainty relation $\Delta N \Delta \varphi \gtrsim 1$. At $x = x_c$, a compressible superfluid turns into an incompressible Mott insulator. Such an insulator tends to maintain a fixed number of particles in a given area, and, at some doping $x < x_c$, the CuO_2 lattice symmetry typically is broken in favor of a superlattice with a large unit cell, tied to $1/x \gg 1$. In this Letter I examine this idea.

The first step is to recognize that the pseudogap physics *differs* in an essential way from the above ^{4}He analogy: cuprates are d-wave superconductors and, in contrast to ^{4}He or s-wave systems, any useful description must contain not only the bosons (Cooper pairs) but also *fermionic* excitations in the form of nodal Bogoliubov–de Gennes (BdG) quasiparticles. The quasiparticles carry well-defined *spin* $S = \frac{1}{2}$, and their coupling to the charge sector, dominated by the $S = 0$ Cooper pairs, is arguably the crucial element of quantum dynamics of cuprates. This spin-charge interaction is topological in origin and peculiar for fluctuating spin-singlet superconductors [8].

The nodal fermions convey additional fundamental information: Cooper pairs in cuprates are inherently the *momentum-space* objects in contrast to the *real-space* pairs behaving as "elementary" bosons, like ^{4}He or the SO(5) hard-core plaquette bosons [4]. Thus, one encounters in cuprates an echo of the historical debate on Blatt-Schafroth versus BCS pairs. This is an important issue—while certain long-distance features of the two descriptions are equivalent, many crucial physical properties are not. In particular, the observed charged modulation is a finite wave vector, nonuniversal phenomenon. As shown in this Letter, the modulation patterns and stable states arising from the two descriptions are essentially different.

To appreciate this difference, note that Cooper pairs in nodal d-wave superconductors are highly nonlocal objects in real space and any description in terms of their center-of-mass coordinates will reflect this nonlocality through complicated intrinsically multibody, extended-range interactions. Such complexity haunts any attempt at constructing a theory using Cooper pairs as elementary bosons. The basic idea advanced in this Letter is that, under these circumstances, the role of elementary bosons should be accorded to *vortices* instead of Cooper pairs. Vortices in cuprates, with their small cores, are simple real-space objects, and the effective theory of quantum fluctuating vortex-antivortex pairs can be written in a form that is local and simple to analyze.

I start by proposing that the modulation observed in [1–3] reflects the Copper pair charge-density wave (CPCDW) in a fluctuating nodal d-wave superconductor. I then show that the physics behind CPCDW relates to an Abrikosov-Hofstadter problem [15,16] for a *dual* type-II lattice superconductor with a flux per unit cell $f = (1 - x)/2$. This mapping allows one to identify stable states as a function of x and to extract the periodicity and orientation of CPCDW relative to the CuO_2 lattice. I elucidate the origin of stable fractions and contrast the results with those for the real-space pairs. The two differ in a fundamental way, akin to the difference between strongly type-II and strongly type-I superconductors. Next, I argue that the formation of CPCDW is *irrelevant* for the physics of nodal fermions—CPCDW is a "high-energy" phenomenon in the parlance of the effective theory [8,9]. Consequently, the leading behavior of nodal fermions remains undisturbed. Finally, I suggest an observable effect of CPCDW which probes an essential element of the theory: the presence of a gauge field which frustrates the propagation of spin, exclusively carried by nodal quasiparticles. The effect is an *enhanced modulation,* with the periodicity related to CPCDW, of the subleading, T^2, term in the spin susceptibility χ. This effect takes place in the "supersolid" state, where superconductivity and CPCDW coexist, and its experimental observation would provide direct evidence of the topological coupling between the fluctuating vortex-antivortex pairs responsible for CPCDW and the electronic *spin*.

The theory of a quantum fluctuating d-wave superconductor was derived in [8] and represents the interactions of fermions with vortex-antivortex excitations in terms of gauge fields v_μ and a_μ:

$$\mathcal{L} = \bar{\Psi}\left[D_0 + iv_0 + \frac{(\mathbf{D}+i\mathbf{v})^2}{2m} - \mu\right]\Psi - i\Delta\Psi^T\sigma_2\hat{\eta}\Psi + \text{c.c.} + \mathcal{L}_0, \tag{1}$$

where $\bar{\Psi} = (\bar{\psi}_\uparrow, \bar{\psi}_\downarrow)$, $\mu = (\tau, x, y)$, $D_\mu = \partial_\mu + ia_\mu\sigma_3$, σ_i's are the Pauli matrices, and $\hat{\eta} \equiv D_x^2 - D_y^2$. $\mathcal{L}_0[v, a]$ arises from the Jacobian of the transformation from discrete vortex coordinates to continuous fields v and a:

$$\int \mathcal{D}[\Phi, \phi_s, A_d, \kappa] C^{-1} e^{\{\int d^3x[i2A_d(\partial\times v) + i2\kappa(\partial\times a) - \mathcal{L}_d]\}}, \tag{2}$$

where $\mathcal{L}_d[\Phi, A_d, \kappa]$ is a dual Lagrangian,

$$\mathcal{L}_d = m_d^2|\Phi|^2 + |(\partial - i2\pi A_d)\Phi|^2 + \frac{g}{2}|\Phi|^4 + |\Phi|^2(\partial\phi_s - 2\pi\kappa)^2, \tag{3}$$

and $C[|\Phi|]$ is a normalization factor,

$$C = \int \mathcal{D}[a, \phi_s, \kappa] e^{\{\int d^3x[i2\kappa(\partial\times a) + |\Phi|^2(\partial\phi_s - 2\pi\kappa)^2]\}}. \tag{4}$$

The physics behind (1) is simple: The fermionic part of $\mathcal{L}$ is just the BdG action for a nodal d-wave superconductor, the awkward phase factor $\exp(i\varphi(x))$ having been removed from Δ by a gauge transformation. This transformation generates gauge fields v and a, which mimic the effect of vortex-antivortex pair fluctuations on the BdG quasiparticles—v in the charge and a in the spin channel. Finally, a bosonic field Φ describes quantum vortex-antivortex pairs, which can be thought of as particles-antiparticles created and annihilated by dual field Φ. The dual "normal" state ($\langle\Phi\rangle = 0$) is a physical superconductor while dual condensate ($\langle\Phi\rangle \neq 0$) describes the pseudogap state. The purpose behind the mathematics is to reformulate the problem in terms of the BdG action for fermions (1) and the local Lagrangian of vortex bosons $\mathcal{L}_d$ (3), kept in mutual communication via two pairs of gauge fields (v, a) and (A_d, κ).

Within this reformulation, the CPCDW, an intractably nonlocal problem in the language of electrons, has a simple local expression in the dual language of vortex field Φ. To recognize this, observe that $\varphi(x)$ couples in $\mathcal{L}$ (1) and (2) to the overall electron density as $\frac{i}{2}\bar{n}\partial_\tau\varphi$, where $\bar{n} = \bar{n}_\uparrow + \bar{n}_\downarrow$. This translates to a dual "magnetic field" $\nabla\times\mathbf{A}_d = \frac{1}{2}\bar{n}$ [7] seen by vortex-antivortex pairs. This effect gives dual voice to the physics discussed earlier: to prevent superfluid ground state the system turns into an incompressible solid, a dual Abrikosov lattice [7,16]. Therefore, the quantum vortex-antivortex unbinding leads to the breaking of (lattice) translational symmetry. When the pattern of symmetry breaking is determined by the dual problem, the results are "communicated" back to the fermionic part of $\mathcal{L}$ (1) via the gauge fields (v, a) and (A_d, κ)—hence CPCDW.

The above arguments are explicit in the dual mean-field approximation, combined with the linearization of the spectrum near the nodes. The linearization splits the fermions in (1) into low-energy nodal spin-$\frac{1}{2}$ Dirac-like particles $\psi_{\sigma,\alpha}$, where $\alpha = 1, \bar{1}, 2$, and $\bar{2}$, and high-energy antinodal fermions combined into spin-singlet Cooper pairs, $\psi_{\sigma,\langle\alpha\beta\rangle}$, where $\langle\alpha\beta\rangle = \langle 12\rangle, \langle 2\bar{1}\rangle, \langle\bar{1}\,\bar{2}\rangle$, and $\langle\bar{2}1\rangle$. Nodal Dirac fermions have no overall charge density—the overall charge is carried by $\psi_{\sigma,\langle\alpha\beta\rangle}$ (Cooper pairs). Furthermore, $\psi_{\sigma,\langle\alpha\beta\rangle}$ form spin singlets and do not couple to a. This enables us to separate the mean-field equations for the spin sector from those for charge:

$$\begin{aligned} \pi\langle n_\uparrow(\mathbf{r},\tau) + n_\downarrow(\mathbf{r},\tau)\rangle &= \nabla\times\mathbf{A}_d(\mathbf{r}), \\ \pm\partial_{y(x)}\delta v_0(\mathbf{r}) &= \pi\mathbf{j}^\Phi_{x(y)}(\mathbf{r}), \\ m_d^2\Phi - (\nabla - i\mathbf{A}_d)^2\Phi + g|\Phi|^2\Phi &= 0, \\ \left\langle\frac{\delta\mathcal{L}}{\delta\Delta(\mathbf{r})}\right\rangle &= (2/\lambda_{\text{eff}})\Delta(\mathbf{r}), \end{aligned} \tag{5}$$

where $n_\sigma(x) = \bar{\psi}_{\sigma,\langle\alpha\beta\rangle}(x)\psi_{\sigma,\langle\alpha\beta\rangle}(x) + \bar{\psi}_{\sigma,\alpha}\psi_{\sigma,\alpha}$, j^Φ_μ is a dual current, $j^\Phi_\mu = -i\Phi^*\partial_\mu\Phi + \text{c.c.} + A_{d\mu}|\Phi|^2$, and λ_{eff} is the effective coupling constant (the last equation is the BdG self-consistency condition for the pseudogap).

The first of Eqs. (5) is an implicit expression for $\delta v_0(\mathbf{r})$. In cuprates, the loss of superconductivity through underdoping is caused by Mott correlations forcing the system into incompressibility. This suggests that the Fourier transform of fermionic compressibility χ_c at the reciprocal lattice vector of the charge modulation is small: $\chi_c(\mathbf{G}) \sim x \ll 1$. Thus, to a good approximation, $\langle\delta n(\mathbf{r})\rangle \approx \chi_c\delta v_0(\mathbf{r})$. From the first Eq. (5) $\chi_c\delta v_0(\mathbf{r}) = \frac{1}{\pi}\nabla\times\mathbf{A}_d(\mathbf{r}) - \bar{n}$, and I recast the next two as

$$\nabla\times(\nabla\times\mathbf{A}_d(\mathbf{r})) = \pi^2\chi_c\mathbf{j}^\Phi(\mathbf{r}), \tag{6}$$

$$m_d^2\Phi - (\nabla - i\mathbf{A}_d(\mathbf{r}))^2\Phi + g|\Phi|^2\Phi = 0. \tag{7}$$

Equations (6) and (7) are the Maxwell and Ginzburg-Landau equations for a type-II dual superconductor [16] in a dual magnetic field $H_d = \pi\bar{n}$ ($\kappa_d \sim 1/\sqrt{\chi_c} > 1/\sqrt{2}$, since χ_c is small for low x).

Equations (6) and (7) are solved as follows [17]: first, various derivatives in (6) and (7) are replaced by their CuO_2 lattice counterparts. We then consider (7) with a uniform dual field $H_d = \frac{1}{2}\bar{n}$ and initially set $g \to 0$. This is a variant of the Hofstadter problem for dual bosons Φ_i, with a fractional flux $f = p/q = (1-x)/2$ through a plaquette of a dual lattice. The solution is a "Hofstadter butterfly" spectrum with deep energy minima for major fractions [15]. The ground state is q-fold degenerate, and one must choose the linear combination of states for dual bosons to condense into. The degeneracy is lifted by finite g in (7). Thus, a unique state $\Phi_i^{(0)}$ is selected, the only remaining degeneracy associated with discrete lattice symmetries. Once $\Phi_i^{(0)}$ is known, one computes the current $\mathbf{j}^\Phi$ and uses Maxwell Eq. (6) to find the modulation in dual induction $\delta\mathbf{B}_d = \mathbf{B}_d - \mathbf{H}_d = \nabla \times \delta\mathbf{A}_d(\mathbf{r})$. This procedure is then iterated to convergence.

The major fractions and their modulation patterns are *primarily* determined by the Abrikosov-Hofstadter problem (7), the magnetic energy being a small correction in a type-II system. The interactions among vortices in $\Phi_i^{(0)}$ responsible for these patterns are intrinsically multibody and of extended range—they are the interactions among the center-of-mass coordinates of Cooper pairs. This is in contrast to the real-space pairs with pairwise short-ranged interactions $V(\mathbf{r}-\mathbf{r}')$. The pair density wave is determined not by (7) but by the Wigner crystallization, encoded in (6), which in this limit turns to the minimization of $\frac{1}{2}\int d^2r d^2r' \mathbf{B}_d(\mathbf{r}) V(\mathbf{r}-\mathbf{r}') \mathbf{B}_d(\mathbf{r}')$, where $\mathbf{B}_d(\mathbf{r}) = \sum_i \delta(\mathbf{r}-\mathbf{r}_i)$ and $\{\mathbf{r}_i\}$ are the pairs' positions. Thus, the Cooper and the real-space pairs correspond to the two *opposite* limits of (6) and (7), that of the type-II and the type-I regimes of a dual superconductor, respectively.

While the analysis of (6) and (7) is given in [17], I outline here general features of the solution. $\Phi_i^{(0)}$ and $\nabla \times \delta\mathbf{A}_d(\mathbf{r})$ break the translational symmetry of the dual and the CuO_2 lattices. The new superlattice is characterized by the set of reciprocal vectors $\{\mathbf{G}_i\}$. The major fractions f, i.e., the energetically most favored states, are those with q being a small integer, (integer)^2 or a multiple of 2, reflecting the square CuO_2 lattice. In the window of x relevant to cuprates, these are $f = 7/16$, 4/9, 3/7, 6/13, 11/24, 15/32, 13/32, 29/64, 27/64, ..., [$x = 0.125(1/8)$, 0.111(1/9), 0.143(1/7), 0.077(1/13), 0.083(1/12), 0.0625(1/16), 0.1875(3/16), 0.09375(3/32), 0.15625(5/32)], etc. Other potentially prominent f, like 1/4, 1/3, 2/5, or 3/8, correspond to x outside the regime of vortex-antivortex fluctuations.

The above information allows insight into $\{\mathbf{G}_i\}$'s of major fractions. The nonlinear term in (7) favors the smallest unit cell containing an integer number of flux quanta and a homogeneous modulation in $|\Phi_i|$. These conditions single out doping $x = 0.125$ ($f = 7/16$) as a particularly prominent fraction. At $x = 0.125$ ($q = 16$), the modulation in $\nabla \times \mathbf{A}_d(\mathbf{r})$ can take advantage of a 4×4 elementary checkerboard block which, when oriented along the $x(y)$ direction, fits neatly into plaquettes of the dual lattice. Near $f = 1/2$, however, a large number of vortices ($p = 7$) per such a block leads to a redistribution and a larger, rhombic unit cell [17]—the energy gain relative to the 4×4 checkerboard, however, is *extremely* small. The modulation in $\delta\bar{n}(\mathbf{r})$ (and Δ) (5) still retains a memory of the 4×4 block and is characterized by wave vectors $\mathbf{G_1} = (\pm 2\pi/4a, 0)$, $\mathbf{G_2} = (\pm 2\pi/8a, \pm 2\pi/4a)$, and $\mathbf{G_3} = (\pm 2\pi/(8a/3), \pm 2\pi/4a)$, with $\mathbf{G_1}$ oriented along the *antinodal* (either x or y) directions of the CuO_2 lattice. The domains of the above modulation pattern offer a natural explanation for the observations in Ref. [2].

The next leading fractions are $x = 0.077(1/13)$ ($f = 6/13$) and $x = 0.111(1/9)$ ($f = 4/9$). The modulation patterns are now more complicated and do not fit easily into the underlying CuO_2 lattice. $\delta\bar{n}(\mathbf{r})$ (and Δ) [17] exhibits a rhombic unit cell with $\{\mathbf{G}_i\}$'s oriented closer to the lattice diagonals, i.e., the *nodal* directions. Thus, as x decreases away from 1/8 there is a tendency to *reorient* the superlattice away from antinodal directions and align it closer to the CuO_2 lattice diagonals. Such reorientation effects of the CPCDW, if observed, would provide support for the physics described in this Letter.

The above considerations include dopings like $x = 1/8$ or 1/9 for which cuprates are typically still superconducting. In such cases the mean field (5) is inadequate, and one must include fluctuations in Φ and A_d. The fluctuations act to depopulate the mean-field ground state $\Phi^{(0)}$ and transfer some of the dual bosons to the states nearby in energy. As x increases toward x_c, $\Phi^{(0)}$ eventually ceases to be *macroscopically* occupied ($\langle\Phi\rangle \to 0$) and the system returns to the superconducting state. However, as long as the transition is not strongly first order, dual bosons preferentially occupy the states close to $\Phi^{(0)}$ on the Hofstadter butterfly energy landscape. This results in $\langle|\Phi(\mathbf{r})|^2\rangle$ which is finite and still modulated. Only for yet higher x will the translational symmetry of the superconductor be finally restored.

The above is an example of the "supersolid state," in which superconductivity coexists with the CPCDW. The modulation is dominated by $\Phi^{(0)}$ and thus our mean-field symmetry analysis of major fractions still goes through. The fluctuations that produce the supersolid state consist of a liquid of vacancies and interstitials superimposed on the original mean-field dual vortex lattice. This leads to low ρ_s and tends to shift the periodicity away from the mean-field set of $\{\mathbf{G}_i\}$'s associated with major fractions, particularly as a function of T, since the self-energies of vacancies and interstitials are generically different. Such fluctuation-induced incommensurability could be behind the difference between the CPCDW periodicities observed in [1] (high T) and [2] (very low T).

The preceding discussion of the charge sector sets the stage for the question of what happens to *spin,* carried by nodal quasiparticles (for convenience, I now rotate a $d_{x^2-y^2}$-wave superconductor into a d_{xy}-wave one). The

CPCDW affects low-energy fermions in two ways: first, $\delta v_0(\mathbf{r})$ couples to $\psi_{\sigma,\alpha}$ as a periodically modulated chemical potential and can be absorbed into a locally varying Fermi wave vector, $k_F \to k_F + \delta k_F(\mathbf{r})$, where $\delta k_F(\mathbf{r}) = \delta v_0(\mathbf{r})/v_F$. Such a shift leaves the nodal point in the energy space undisturbed. Similarly, there also is a modulation in the size of the pseudogap, $\Delta \to \Delta + \delta\Delta(\mathbf{r})$, arising from the BdG self-consistency Eq. (5). Near the nodes $\Delta(\mathbf{P};\mathbf{k}) \to \Delta(\hat{k}_x^2 - \hat{k}_y^2) + \delta\Delta(\{\mathbf{G}_i\},\mathbf{k})$, where $\mathbf{P}$ is related to the center-of-mass momentum of Cooper pairs. Assuming that the pseudogap retains the overall $d_{x^2-y^2}$-wave symmetry throughout the underdoped regime, one finds $\delta\Delta(\{\mathbf{G}_i\},\mathbf{k}) \sim \hat{k}_x^2 - \hat{k}_y^2$. Again, the nodal point is left intact. The semiclassical spectrum is

$$E(\mathbf{k};\mathbf{r}) = \pm\sqrt{v_F^2(\mathbf{r})k_x^2 + v_\Delta^2(\mathbf{r})k_y^2}, \tag{8}$$

where $v_F(\mathbf{r}) = v_F + (\delta k_F(\mathbf{r})/m)$ and $v_\Delta(\mathbf{r}) = v_\Delta + (\delta\Delta(\mathbf{r})/k_F)$ [18]. The local DOS exhibits modulation at wave vectors $\{\mathbf{G}_i\}$'s but still vanishes linearly at the nodes. The only exception to this behavior is the situation in which CPCDW is *commensurate* with the nodes and $\{\mathbf{G}_i\}$'s happen to coincide with some of the internodal wave vectors: $\mathbf{Q}_{1\bar{1}}$, $\mathbf{Q}_{1\bar{2}}$, etc. Such commensuration can only be purely accidental since the dual lattice physics (6) and (7) that determines $\{\mathbf{G}_i\}$'s has no simple relation to the location of nodes in the Brillouin zone.

There is, however, yet another way by which the CPCDW affects nodal fermions $\psi_{\sigma,\alpha}$. This is through the coupling to a Berry gauge field a_μ, which describes topological frustration of BdG "spinons" moving through space filled with fluctuating $hc/2e$ vortex-antivortex pairs. This nontrivial coupling of charge and spin sectors is captured by the effective Lagrangian,

$$\mathcal{L}_{\mathrm{f}} = \bar{\psi}_n(i\gamma_\mu\partial_\mu + \gamma_\mu a_\mu)\psi_n + \mathcal{L}_0^a[a_\mu], \tag{9}$$

obtained as the low-energy ($\ll\Delta$) limit of $\mathcal{L}$ (1). In (9), $\psi_{\sigma,\alpha}$ have been arranged into four component Dirac-BdG spinors ψ_n following conventions of Ref. [8] and the summation over $N = 2$ nodal flavors is understood.

Below the pseudogap energy scale Δ, the spin correlator of *physical* electrons is

$$\langle S_z(-k)S_z(k)\rangle = \frac{\Pi_A^F(k)\Pi_A^0(k)}{\Pi_A^F(k) + \Pi_A^0(k)}\frac{\mathbf{k}^2}{\mathbf{k}^2 + \omega_n^2}, \tag{10}$$

where $\Pi_A^F(k) \sim |k|$ denotes the fermion current polarization and Π_A^0 is the bare stiffness of a in $\mathcal{L}_0$. In the pseudogap state a is massless and $\Pi_A^0 \sim k^2$ dominates the expression for the static spin susceptibility χ, leading to a non-Fermi liquid behavior of nodal quasiparticles. In the superconducting state, a has mass M^2, $\Pi_A^0 \to M^2$, and the leading order behavior is set by Π_A^F. For $T \ll \Delta$,

$$\chi \sim (2N\ln 2/\pi)T - \frac{(2N\ln 2/\pi)^2}{M^2}T^2 + \ldots. \tag{11}$$

The leading term ($\sim T$) in (11) is just the renormalized d-wave Yoshida function of noninteracting BdG quasiparticles. The subleading term ($\sim T^2$), however, involves M^2. In the supersolid phase M^2 is *modulated* via the nonuniformity in $\langle|\Phi|^2\rangle$ (3)—this modulation carries an imprint of the CPCDW periodicity set by $\{\mathbf{G}_i\}$'s, since it reflects the variation of $\langle|\Phi|^2\rangle$ on the lattice dual to the CuO_2 one. Furthermore, since $M^{-2} \sim \xi_{\mathrm{d}}$ [8], where ξ_{d} is the *dual* superconducting correlation length, the T^2 term in (11) is $\propto \xi_{\mathrm{d}}$ and consequently strongly enhanced as $x \to x_c$. The combination of modulation and enhancement, as the superconductivity is extinguished at $x = x_c$, sets this term apart from other contributions to χ. The observation of such a modulation, in a muon spin relaxation or an NMR experiment, for instance, would provide a vivid illustration of the subtle interplay between the charge and spin channels which is the hallmark of theory (1).

This work was supported in part by the NSF Grant No. DMR00-94981.

[1] M. Vershinin *et al.*, Science **303**, 1995 (2004).

[2] K. McElroy *et al.*, cond-mat/0404005; J. E. Hoffman *et al.*, Science **295**, 466 (2002).

[3] C. Howald *et al.*, Phys. Rev. B **67**, 014533 (2003).

[4] H. D. Chen *et al.*, Phys. Rev. Lett. **93**, 187002 (2004); H. D. Chen *et al.*, Phys. Rev. B **70**, 024516 (2004).

[5] H. C. Fu, J. C. Davis, and D. H. Lee, cond-mat/0403001.

[6] V. J. Emery and S. A. Kivelson, Nature (London) **374**, 434 (1995).

[7] L. Balents, M. P. A. Fisher, and C. Nayak, Phys. Rev. B **60**, 1654 (1999).

[8] M. Franz and Z. Tešanović, Phys. Rev. Lett. **87**, 257003 (2001); O. Vafek and Z. Tešanović, *ibid.* **91**, 237001 (2003); M. Franz, Z. Tešanović, and O. Vafek, Phys. Rev. B **66**, 054535 (2002).

[9] I. F. Herbut, Phys. Rev. Lett. **88**, 047006 (2002).

[10] S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).

[11] Y. Zhang, E. Demler, and S. Sachdev, Phys. Rev. B **66**, 094501 (2002).

[12] J. Zaanen and O. Gunnarson, Phys. Rev. B **40**, 7391 (1989); S. A. Kivelson *et al.*, Rev. Mod. Phys. **75**, 1201 (2003).

[13] L. Reatto and G. V. Chester, Phys. Rev. **155**, 88 (1967).

[14] M. P. A. Fisher *et al.*, Phys. Rev. B **40**, 546 (1989).

[15] D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976).

[16] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).

[17] A. Melikyan and Z. Tešanović, cond-mat/0408344.

[18] The reasoning behind (8) can be fortified beyond semiclassical: the modulation $\delta v_0(\mathbf{r})$ defines some new "band structure" of a lattice d-wave superconductor with an effective hopping t_{ij} and a bond pseudogap Δ_{ij}. If we fold the original Brillouin zone to accommodate the supercell of charge modulation, the nodal points are still at the new Fermi surface, as long as Δ_{ij} retains the overall d-wave symmetry. Near the nodes, the spectrum is linear with perturbative velocity renormalizations $\sim(\delta v_0(\{\mathbf{G}\}_i))^2$.