

Quasiparticle spectra in the vicinity of a d -wave vortex

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We discuss the evolution of the local quasiparticle spectral density and the related tunneling conductance measurable by the scanning tunneling microscope, as a function of distance r and angle θ from the vortex core in a $d_{x^2-y^2}$ superconductor. We consider the effects of electronic disorder and of a strongly anisotropic tunneling matrix element, and show that in real materials they will likely obscure the $\sim 1/r$ asymptotic tail in the zero-bias tunneling conductance expected from the straightforward semiclassical analysis. We also give a prediction for the tunneling conductance anisotropy around the vortex core and establish a connection to the structure of the tunneling matrix element. [S0163-1829(99)11329-8]

I. INTRODUCTION

While there remains almost no doubt at present that the hole-doped high- T_c cuprate superconductors possess an unconventional $d_{x^2-y^2}$ order parameter, the microscopic origin and many phenomenological consequences of this fact remain to be understood. The situation is most pressing in the presence of applied external magnetic field where the interplay between the spatially varying order parameter, supercurrents, and low energy quasiparticles results in a great variety of novel effects. In the Meissner state Yip and Sauls had predicted a nonlinear Meissner effect^{1,2} manifested by an anisotropic component of the in-plane penetration depth linear in field, a unique consequence of the nodal structure of the d -wave order parameter. However, despite considerable experimental effort, this effect has not been clearly observed in cuprates³ and, very recently, Li, Hirschfeld, Wölfle⁴ argued that the effect might not be observable in cuprates, even in principle, due to the highly nonlocal nature of the electromagnetic response of a d -wave superconductor at low temperatures (see, however, Ref. 5). In the mixed state similar nonlocal effects have been predicted to result in a very rich equilibrium vortex lattice structure phase diagram,⁶ but again, no conclusive experimental confirmation has yet been reported.

Among the more successful theoretical predictions specific to the d -wave order parameter is Volovik's prediction of a $\sim T\sqrt{H}$ contribution to the specific heat⁷ which was identified in measurements on $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) single crystals.⁸ Although subsequent experimental investigations reported deviations from the precise \sqrt{H} behavior,⁹ they were consistent with more general scaling relations^{10,11} based on the same general physical picture. In its simplest form the $\sim T\sqrt{H}$ behavior can be derived by observing that the supercurrent orbiting each individual vortex Doppler-shifts the local quasiparticle spectrum, $E_{\mathbf{k}} \rightarrow E_{\mathbf{k}} - \mathbf{v}_s(\mathbf{r}) \cdot \mathbf{k}$, which in turn results in finite residual density of states $N(E=0, \mathbf{r}) \propto v_s(\mathbf{r}) \propto 1/r$. Integrating $N(0, \mathbf{r})$ up to the intervortex distance $R_H \propto 1/\sqrt{H}$ one obtains the total density of states per vortex and the Volovik's result then follows on multiplying by the number of vortices $n_v = H/\Phi_0$, with Φ_0 the flux quantum. We note that behavior consistent with the \sqrt{H} dependence of the

residual density of states was found in numerical calculations within the Bogoliubov–de Gennes (BdG) formalism.¹²

Here we wish to theoretically address the surprising fact that the expected $N(0, \mathbf{r}) \propto 1/r$ dependence of the local density of states (LDOS) has not been observed in the scanning tunneling spectroscopy (STS) measurements,^{13–15} despite the fact that this state of the art technique definitely possesses the required spatial and energy resolution. Inspection of the data reveals that instead of a $1/r$ asymptotic tail at large distances the spectra for YBCO and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ (BSCCO) recover their zero-field profiles within short distances from the cores on the order of several coherence lengths, beyond which the spectra remain unchanged. There are several compelling reasons why it is desirable to resolve this potential conflict between the STS and thermodynamic measurements. The most important one has to do with confirming the picture of a well defined d -wave quasiparticle in the superconducting state of cuprates. In particular, since it is generally believed that the normal state of the cuprates is not a conventional Fermi liquid, it is of considerable importance to verify, in as exhaustive detail as possible, that the ordered state below T_c , where the Fermi-liquid scenario is believed to apply, indeed exhibits all the expected features predicted by the theory. When this fundamental issue has been clarified one can perhaps hope to tackle greater problems in the field, such as the nature of the non-Fermi-liquid-like behavior above T_c and the origin of the pairing mechanism.

In order to achieve our goal we first demonstrate, by direct comparison to the results of fully self-consistent BdG theory for a single vortex, that the semiclassical Volovik approach indeed captures the right physics of single-particle excitations *outside* the vortex core. We then proceed to extend this approach to the realistic case of quasiparticles with finite lifetimes and with strongly anisotropic c -axis tunneling matrix element. Our main result is that the abovementioned STS data away from the vortex core can be understood by considering the effect of the matrix element $M_{\mathbf{k}}$ for the electron tunneling along the c axis. According to the band structure considerations for tetragonal cuprates $M_{\mathbf{k}}$ exhibits the same anisotropy as the gap function, i.e., it vanishes linearly on the Fermi surface near the zone diagonals.^{16,17} In such a case the tunneling conductance $g(V, \mathbf{r})$ is not simply proportional to the temperature-broadened LDOS, but reflects the

additional structure in $M_{\mathbf{k}}$. Analysis of a case when $M_{\mathbf{k}} \propto \cos 2\phi$ leads to a surprising conclusion that the power law in the decay of the zero-bias tunneling conductance $g(0, \mathbf{r})$ changes to $1/r^3$, meaning that it vanishes much faster than the expected $1/r$ decay for LDOS. We argue that combined with the effects of electronic disorder, which tends to further wash out the effect of supercurrents on $g(0, \mathbf{r})$, this mechanism is responsible for the absence of the $1/r$ conductance tail observed in STS, *despite* the fact that LDOS itself exhibits the $1/r$ behavior.

We further show that there exists a direct relationship between the anisotropy of $M_{\mathbf{k}}$ and the real-space anisotropy of $g(V, \mathbf{r})$ around the vortex core. Although we find that the simple semiclassical approach employed in this work *does not* capture the details of the angular distribution of $g(V, \mathbf{r})$ around the vortex, we expect the latter conclusion regarding the matrix element effect to have rather general validity. On the conceptual front our findings illustrate the strengths and the limitations of the Volovik-type approach.

II. TUNNELING CONDUCTANCE: GENERAL FORMALISM

A. Zero field

Tunneling conductance at bias V between a superconductor and a normal metal is given by

$$g(V) = - \int_{-\infty}^{\infty} d\omega f'(\omega - eV) \sum_{\mathbf{k}} |M_{\mathbf{k}}|^2 A(\mathbf{k}, \omega), \quad (1)$$

where f is the Fermi function and $A(\mathbf{k}, \omega) = -2\text{Im}\mathcal{G}(\mathbf{k}, \omega)$ is the spectral function of a superconductor related to the diagonal part of the full superconducting Greens function $\hat{G}(\mathbf{k}, \omega)$. In quasi-two-dimensional (2D) cuprates the spectral function is taken to describe electrons within a single ab -plane and, correspondingly, \mathbf{k} refers to a 2D wave vector. While Eq. (1) could be written down on purely intuitive grounds a more detailed discussion of how one actually implements this dimensional reduction is given in the Appendix.

For the clean system the formulation based on Eq. (1) is fully equivalent to the analogous expression in terms BdG wave functions but allows for a straightforward inclusion of the effects of finite quasiparticle lifetime. In the absence of field the diagonal Greens function assumes the well known form¹⁸ (taking $\hbar = 1$)

$$\mathcal{G}(\mathbf{k}, \omega) = \frac{(\omega - i\Gamma) + \epsilon_{\mathbf{k}}}{(\omega - i\Gamma)^2 - \epsilon_{\mathbf{k}}^2 - \Delta_{\mathbf{k}}^2}, \quad (2)$$

where $\epsilon_{\mathbf{k}}$ is the normal state electron dispersion, $\Delta_{\mathbf{k}} = \Delta_d \cos 2\phi$ is the d -wave gap, and Γ models the quasiparticle lifetime broadening that results from random disorder and inelastic processes. In a d -wave superconductor, strictly speaking, the lifetime effects should be described by a frequency and wave-vector-dependent self-energy $\Sigma(\mathbf{k}, \omega)$ whose precise structure, however, is not well understood at present. Since we are mainly interested in the effect of vortices on the spectral properties at the lowest energies, we shall in the following ignore this complication and simply parameterize the lifetime effects by a constant scattering rate

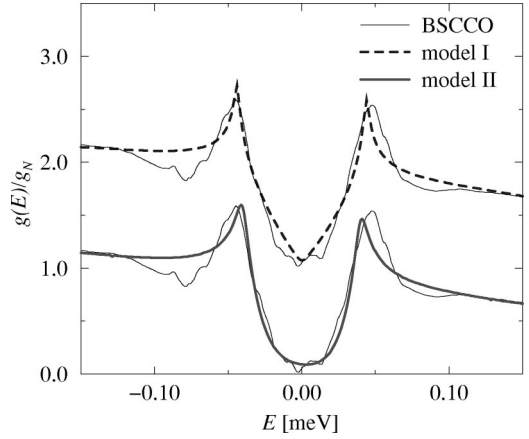


FIG. 1. Best fit of tunneling conductance from Eq. (1) to the zero-field experimental data on underdoped BSCCO of Renner *et al.* (Ref. 20), for models I and II. The fitted parameters were $\Delta_d = 41.0$ and $\Gamma = 1.4$ meV for model I and $\Delta_d = 39.7$ and $\Gamma = 3.9$ meV for model II. A term linear in E has been added to Eq. (1) in order to account for the background conductance and the data for model I are offset for clarity.

Γ . We expect this approximation to be entirely adequate in the present context since the important qualitative features of the tunneling conductance discussed below emerge clearly in the clean limit $\Gamma = 0$.

As alluded to in the Introduction, nontrivial dependence of the matrix element $M_{\mathbf{k}}$ on the angle ϕ of the \mathbf{k} vector on the Fermi surface has measurable consequences for the tunneling conductance in an anisotropic superconductor. While the motivation underlying this idea is primarily phenomenological, we discuss below a possible microscopic justification for the matrix element anisotropy within a class of models that assume coherent interlayer transport. Band structure considerations in tetragonal cuprates^{16,17} imply strong anisotropy of the *interlayer* tunneling matrix element $t_{\perp}(\mathbf{k}) \propto \cos 2\phi$. One direct consequence of this anisotropy is the well known qualitative difference between the temperature dependences of the in-plane and c -axis penetration depths.¹⁶ It is reasonable to expect that the structure of $t_{\perp}(\mathbf{k})$ will directly translate into similar anisotropy in the matrix element $M_{\mathbf{k}}$ for tunneling between the superconductor and the STS tip. The Appendix confirms this expectation by providing a formal derivation of $M_{\mathbf{k}}$ from the transfer Hamiltonian formulation of the tunneling problem in the planar tunneling geometry. Motivated by these considerations in the following we study two models: a conventional model I with $M_{\mathbf{k}} = M_0$ and an anisotropic model II with $M_{\mathbf{k}} = M_2 \cos 2\phi$, as suggested by Ref. 16, where $\mathbf{k} = (k, \phi)$ in polar coordinates. In model I tunneling conductance is simply proportional to LDOS while in model II tunneling from the zone diagonals is suppressed.

It is straightforward to numerically evaluate Eq. (1) for the two models and compare $g(V)$ to the experimental data. The result of the best fit to the zero-field data on BSCCO (Ref. 20) is displayed in Fig. 1. It is seen that, as pointed out previously in Ref. 19, model II captures the qualitative features of the data much better than model I. The wide, U-shaped conductance near the zero bias appears to be a generic feature of the c -axis tunneling conductance in tetrag-

onal cuprates,^{20–23} and is inconsistent with linearly vanishing $g(V)$ of model I.^{24,25} We therefore conclude that the available tunneling data are consistent with model II, as expected from the band structure argument presented above.

B. Finite field-semiclassical treatment

The effects of applied magnetic field are taken into account by performing a semiclassical replacement

$$\omega \rightarrow \omega - \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r}) \quad (3)$$

in the Greens function of Eq. (2). Here $\mathbf{v}_s(\mathbf{r})$ is the local superfluid velocity which in the vicinity of a single vortex has the form²⁶

$$\mathbf{v}_s(\mathbf{r}) = \frac{\hat{\theta}}{2mr}, \quad (4)$$

and is cut off exponentially at distances in excess of the London penetration depth λ . The semiclassical approximation (3) was at the heart of the original Volovik calculation of the specific heat⁷ and has been used extensively to compute various spectral^{19,27,28} and transport^{29–31} properties of the mixed state. It appears to capture very well the essential physics of d -wave quasiparticles moving on the slowly varying background of the vortex lattice.

We have explicitly verified that, for $r \geq 2\xi$, Eqs. (1)–(4) yield very reasonable LDOS profiles over the entire energy range of interest when compared to the results of a fully self-consistent calculation within the BdG theory for a single d -wave vortex.^{32,33} Figure 2 illustrates the agreement between the two approaches. Note in particular the excellent agreement in the low-energy part of the spectrum. We believe that this comparison constitutes a stringent test for the validity of the semiclassical approximation (3) for the local quantities such as LDOS outside of the vortex core. As expected, however, inside the core the semiclassical approximation breaks down, as visible in the top panel of Fig. 2. In the strongly type-II cuprates at fields well below H_{c2} cores comprise only a small fraction of the total volume. Semiclassical approximation thus works well almost everywhere which explains the success of the Volovik picture in modeling of the mixed state.

With the replacement (3) the tunneling conductance (1) becomes position dependent through the spatial dependence of the superfluid velocity. In the following we discuss the local tunneling conductance $g(V, \mathbf{r})$ near a single isolated vortex in a $d_{x^2-y^2}$ superconductor.

III. TUNNELING SPECTRA IN THE VICINITY OF THE VORTEX

A. Clean limit, zero temperature

In the clean limit $\Gamma \rightarrow 0^+$ the Doppler-shifted spectral function assumes a simple form

$$A(\mathbf{k}, \omega) = \pi \sum_{\nu=\pm 1} \left(1 - \nu \frac{\epsilon_{\mathbf{k}}}{E_{\mathbf{k}}} \right) \delta(\omega - \eta + \nu E_{\mathbf{k}}), \quad (5)$$

where $E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$ and $\eta = \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})$. We now evaluate the corresponding tunneling conductance given by Eq. (1). At

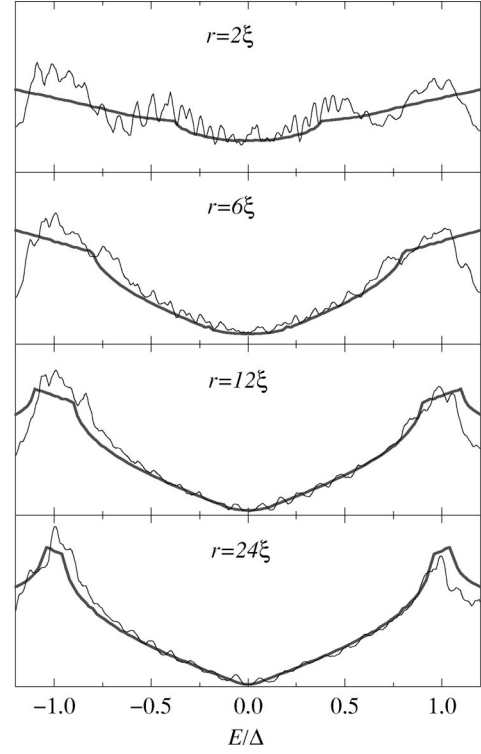


FIG. 2. Comparison of the LDOS obtained using semiclassical approximation Eqs. (1)–(4) with $\Gamma = 0$ (thick lines) and the numerical solution of the BdG equations of Ref. 32 (thin lines) at indicated distances from the core. In both cases averages over the real-space angle θ are plotted.

$T = 0$ the f' factor becomes a δ function and the ω integral is trivial. The remaining sum over \mathbf{k} is replaced by an integral in the usual manner. Assuming the free electron dispersion $\epsilon_{\mathbf{k}} = k^2/2m - \epsilon_F$ we may use the δ function in Eq. (5) to explicitly perform the integral over the energy variable and obtain the tunneling conductance as a Fermi surface average of the form

$$\frac{g(V, \mathbf{r})}{g_N} = \int_0^{2\pi} \frac{d\phi}{2\pi} \text{Re} \left[\frac{(\sqrt{2} \cos 2\phi)^n |eV - \eta|}{\sqrt{(eV - \eta)^2 - \Delta(\phi)^2}} \right]. \quad (6)$$

Here g_N is the normal state conductance, n assumes values of 0 and 2 for models I and II, respectively, and we have restricted \mathbf{k} in η to the Fermi surface. We may thus write

$$\eta = \mathbf{k}_F \cdot \mathbf{v}_s(\mathbf{r}) = \frac{\pi}{2} \Delta_d \frac{\xi}{r} \sin(\theta - \phi), \quad (7)$$

with $\mathbf{r} = (r, \theta)$ and $\xi = v_F / \pi \Delta_d$ the coherence length.

For arbitrary bias V the conductance (6) must be evaluated numerically. However, for $|eV| \ll \Delta_d$ the integral is dominated by the regions close to the four nodes of $\Delta(\phi)$ and can be evaluated, to an excellent approximation, by expanding to leading order in ϕ near the nodes. For the zero-bias conductance we thus obtain

$$\frac{g(0, \mathbf{r})}{g_N} \approx \sum_{l=1}^4 \int_0^{z_l/2} \frac{d\phi}{\pi} (2\sqrt{2}\phi)^n \frac{z_l}{\sqrt{z_l^2 - (2\phi)^2}}, \quad (8)$$

where

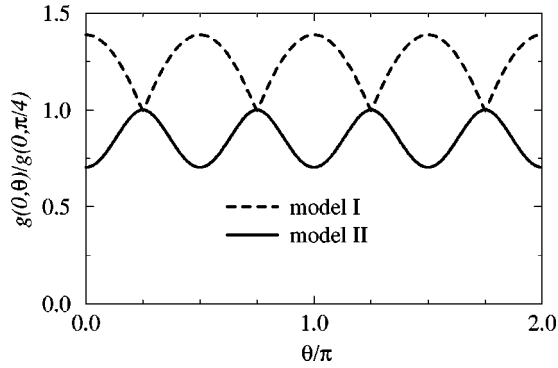


FIG. 3. Zero-bias tunneling conductance as a function of real space polar angle θ around the vortex normalized to unity along the nodal direction $\theta = \pi/4$.

$$z_l = \frac{\pi}{2} \frac{\xi}{r} |\sin(\theta - \phi_l)|, \quad (9)$$

and $\phi_l = \pi(2l-1)/4$ are the nodal points. The integral implied by Eq. (8) is elementary and yields

$$\frac{g(0, \mathbf{r})}{g_N} \simeq c_n \left(\frac{\xi}{r}\right)^{n+1} [|\sin \tilde{\theta}|^{n+1} + |\cos \tilde{\theta}|^{n+1}], \quad (10)$$

where $\tilde{\theta} = \theta - \pi/4$ is the polar angle measured from a node, $c_0 = \pi/4 \simeq 0.78$ and $c_2 = \pi^3/16 \simeq 1.94$.

According to Eq. (10) the symmetry of the tunneling matrix element $M_{\mathbf{k}}$ has profound consequence for the spatial dependence of $g(0, \mathbf{r})$ near the vortex core. Most importantly we notice that the decay with the distance r from the core is much more rapid in model II, where $g(0, \mathbf{r}) \sim (\xi/r)^3$, compared to the ξ/r behavior in model I. For instance, at $r = 3\xi$ the zero-bias conductance will be suppressed by a factor of 3 in model I but by a factor of 27 in model II. We argue that this difference is a very likely reason for the observed absence of $1/r$ tails in STS measurements.

Equation (10) implies an interesting prediction for the angular dependence of $g(0, \mathbf{r})$ for fixed r , corresponding to taking a scan along a circle of the radius r centered at the core. This angular dependence is a result of the underlying \mathbf{k} -space anisotropy of the gap function and is illustrated in Fig. 3 for models I and II. While the ratio between maximum and minimum is $\sqrt{2}$ for both the models, we observe that there is a qualitative difference between the two models in the positions of maxima and minima.

We note that numerical calculations within the Eilenberger^{34,35} and BdG (Refs. 36,37) formalisms show *maxima* of zero-bias LDOS along the $\pi/4$ directions, in contradiction to the above conclusions. This indicates inadequacy of the present semiclassical approach in modeling the fine details of the spatial distribution of LDOS near a vortex. Nevertheless, our result clearly establishes that the matrix element anisotropy will have measurable effect on the tunneling conductance even when included in more elaborate microscopic models. We also emphasize that present approach correctly captures the angle-averaged LDOS, as demonstrated in Fig. 3.

B. Effect of temperature and finite lifetime

It is evident that finite lifetime and temperature will affect the prediction (10) for the tunneling conductance, since they both lead to finite zero-bias conductance even in the absence of field. One expects that these effects become significant at distances from the core where the characteristic Doppler-shift energy $E_D = \Delta_d(\xi/r)$ becomes comparable to Γ or T . At 4.2 K, which is typically the temperature of STS experiment, temperature broadening becomes important at $r/\xi \gtrsim \Delta_d/4.2$ K which is a number of order 100 in cuprates. Thus, for low-temperature tunneling the effects of thermal broadening are unimportant, except at large distances from the cores. Scattering rate Γ , on the other hand, can be a significant fraction of the maximum gap in cuprates, as evidenced by a relatively large zero-bias conductance observed experimentally,^{14,22} and will therefore cause significant broadening at distances of several ξ from the core. We now discuss the effect of finite lifetime Γ in some detail.

For $\Gamma > 0$ the spectral function becomes

$$A(\mathbf{k}, \omega) = \sum_{\nu=\pm 1} \left(1 - \nu \frac{\epsilon_{\mathbf{k}}}{E_{\mathbf{k}}}\right) \frac{\Gamma}{(\omega - \eta + \nu E_{\mathbf{k}})^2 + \Gamma^2}. \quad (11)$$

Evaluation of the \mathbf{k} sum in Eq. (1) is now somewhat more involved since we no longer have a δ function at our disposal to perform the energy integral. At $T=0$ the zero-bias tunneling conductance is

$$g(0, \mathbf{r}) = C \int_0^{2\pi} d\phi (\cos 2\phi)^n \times \int_0^\infty d\epsilon \left[\frac{\Gamma}{(\eta + E)^2 + \Gamma^2} + \frac{\Gamma}{(\eta - E)^2 + \Gamma^2} \right], \quad (12)$$

where $E = \sqrt{\epsilon^2 + \Delta(\phi)^2}$ and C contains all the constant prefactors. We evaluate Eq. (12) by linearizing all the functions in the integrand containing ϕ around the four nodes of the gap function, e.g., $\Delta(\phi - \phi_l) \approx 2\Delta_d \phi$, and extending the angular integration to infinity. Under the assumption that $\Gamma, |\eta| \ll \Delta_d$ the resulting integral can be evaluated by making use of new variables, $u = \sqrt{\epsilon^2 + (2\Delta_d \phi)^2}$ and $\alpha = \arctan[\epsilon/2\Delta_d \phi]$. The α integration is trivial and we obtain

$$\frac{g(0, \mathbf{r})}{g_N} = \frac{1}{4\pi} \sum_{l=1}^4 \int_0^\Lambda du \left(\frac{u}{\Delta_d}\right)^{n+1} \times \left[\frac{\Gamma}{(u + \Delta_d z_l)^2 + \Gamma^2} + \frac{\Gamma}{(u - \Delta_d z_l)^2 + \Gamma^2} \right], \quad (13)$$

where z_l is defined in Eq. (9) and Λ is a cutoff of the order of Δ_d imposed in order to assure convergence at large u .³⁸ Finally, carrying out the integral and keeping in mind that $\Gamma, |\eta| \ll \Delta_d$, we obtain the leading terms in model I,

$$\frac{g(0, \mathbf{r})}{g_N} \simeq \frac{1}{4\pi} \sum_{l=1}^4 \left[2z_l \arctan\left(\frac{z_l}{\gamma}\right) - \gamma \ln(\gamma^2 + z_l^2) \right], \quad (14)$$

and for model II,

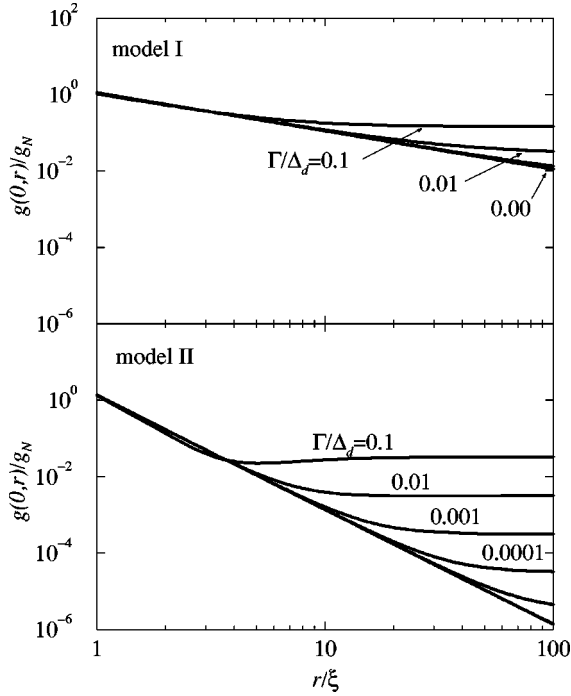


FIG. 4. Effect of finite quasiparticle lifetime Γ on the amplitude of the zero-bias tunneling conductance as a function of distance r from the vortex core (for $\phi=0$). Note the log-log scale. The straight lines with slopes -1 and -3 mark the expected asymptotic behaviors for $\Gamma=0$ in models I and II, respectively.

$$\frac{g(0,\mathbf{r})}{g_N} \approx \frac{1}{4\pi} \sum_{l=1}^4 \left[2z_l(z_l^2 - 3\gamma^2) \arctan\left(\frac{z_l}{\gamma}\right) + \gamma \right]. \quad (15)$$

Here we have set $\Lambda = \Delta_d$ and defined a dimensionless scattering rate $\gamma = \Gamma/\Delta_d$.

Equations (14) and (15) exhibit the correct behavior in the limit $\Gamma \rightarrow 0^+$ when compared to the result for the clean system (10). For finite Γ they describe the crossover from the $1/r^{n+1}$ behavior close to the core ($r \ll r_\Gamma$) to the lifetime dominated constant zero-bias conductance far from the core ($r \gg r_\Gamma$). Inspection of Eqs. (14) and (15) reveals that the dependence of the crossover scale r_Γ on Γ is more subtle than one would expect from the simple argument involving the Doppler-shift energy E_D presented above. In particular we find that

$$r_\Gamma \sim -\xi/(\gamma \ln \gamma) \quad (16)$$

for model I while

$$r_\Gamma \sim \xi/\gamma^{1/3} \quad (17)$$

for model II. In both cases lifetime effects become important at *shorter* distances from the core than one would expect from the naive estimate $r_\Gamma \sim \xi/\gamma$. This is illustrated in Fig. 4 where we plot the crossover functions for the tunneling conductance given by Eqs. (14) and (15) as a function of distance r from the core for a number of lifetimes Γ . We note that in model II lifetime effects are much more efficient in destroying the clean power law behavior than in model I, consistent with Eqs. (16) and (17).

The lifetime effects will also affect the anisotropy of the tunneling conductance around the core, since one would ex-

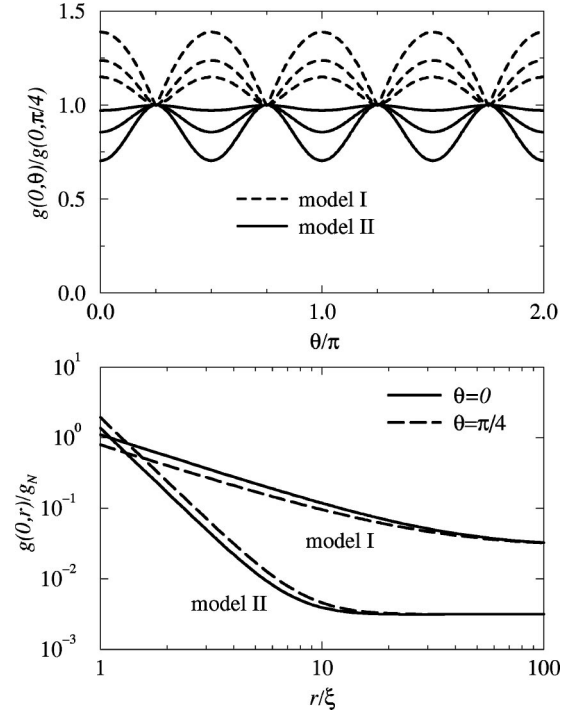


FIG. 5. Effect of finite quasiparticle lifetime Γ on the angular anisotropy of the zero-bias tunneling conductance. Upper panel: angular dependence (normalized to unity at $\theta = \pi/4$) for $\Gamma/\Delta_d = 0.01$ and distances $r/\xi = 1, 10, 20$ in the order of diminishing anisotropy. Lower panel: maximum and minimum conductivities as a function of distance r from the vortex core for $\Gamma/\Delta_d = 0.01$.

pect $g(0,\mathbf{r})$ to become isotropic at distances larger than r_Γ . Figure 5 displays the evolution of the anisotropy as a function of angle θ for increasing distance r at constant Γ (upper panel). Lower panel compares the angular maximum to the minimum as a function of r . As expected, lifetime effects wash out the anisotropy at distances from the core in excess of the crossover scale r_Γ .

IV. SUMMARY AND CONCLUSIONS

Our main objective was to reconcile the apparent absence of the $1/r$ tails in the local tunneling conductance in the vicinity of a d -wave vortex, expected on the basis of a simple Volovik model, with the general consensus that such a semiclassical picture captures the essential physics of the mixed state in cuprates. We argued that the STS measurements are likely dominated by the nontrivial structure of the tunneling matrix element, which may be derived from the band structure considerations within models assuming coherent inter-layer tunneling.^{16,17} We showed that, for the matrix element of the form $M_{\mathbf{k}} = M_2 \cos(2\phi)$ (model II), in the absence of lifetime effects, the zero-bias tunneling conductance power law is modified to $1/r^3$, making it vanish much faster than the $1/r$ tail obtained for constant $M_{\mathbf{k}}$ (model I).

We predicted a substantial angular anisotropy of the zero-bias tunneling conductance, with maximum to minimum ratio of $\sqrt{2}$, and with exactly opposite arrangement of extrema in the two models. Our conclusion regarding the precise form of this anisotropy, however, is somewhat less certain in view of the fact that it disagrees with the numerical results

obtained within various microscopic models.^{34–37} Nonetheless, our result clearly indicates that the structure of the tunneling matrix element will have significant impact on the real-space anisotropy of $g(V, \mathbf{r})$. The reliable information about the details of this effect can be obtained in a relatively straightforward manner by incorporating the nontrivial matrix element into a fully self-consistent BdG calculation along the lines of Ref. 32; the work on this is in progress. Within this method it will also be interesting to study the effect of the matrix element on the tunneling spectra in the vortex core, a problem inaccessible to the semiclassical approximation.

Lifetime effects will cut off both the power law decay and the anisotropy at distances beyond the crossover length scale r_Γ given by Eqs. (16) and (17), where r_Γ is always shorter in model II for a given value of Γ . This means that if model II is the physically relevant one, as it appears to be the case for BSCCO, then the lifetime effects may likely render the experimental detection of the asymptotic $1/r^3$ behavior very difficult. For instance, if we assume $\Gamma/\Delta_d=0.1$, which is physically reasonable for BSCCO at low T (Ref. 19), then Eq. (17) implies $r_\Gamma \approx 2.1\xi$, meaning that there will be virtually no asymptotic region where the $1/r^3$ behavior could be observed. Instead, one would see an onset of the constant zero-bias tunneling conductance just outside of the vortex core. This theoretical conclusion is in fact consistent with the STS data of Renner *et al.*¹⁴ Even in the presence of substantial lifetime effects it should still be in principle possible to observe the predicted angular anisotropy of $g(0, \mathbf{r})$ close to the core. The existing experimental data on BSCCO in fact show a definite hint of a fourfold anisotropy, which is, however, discernible only at high bias.¹⁵ The above analysis suggests that the asymptotic tails might become observable in the cleanest samples characterized by low value of the zero-bias conductance in the absence of field. Clearly, the effect would be difficult to discern in the existing data on YBCO (Ref. 13) which display large zero-bias conductance of unknown origin.

It would thus appear that proper inclusion of the anisotropy in the tunneling matrix element naturally resolves the conflict between the STS and the heat capacity measurements, since the latter is obviously insensitive to the structure of the matrix element. While certain details still await experimental verification, this seems to be a satisfactory tentative conclusion, especially since additional experimental evidence emerged recently that appears to further solidify the support for the Volovik-type description of the mixed state in cuprates. In particular measurements of the complex conductivity (using coherent terahertz spectroscopy) in the mixed state of BSCCO films reported by Malozzi *et al.*³⁹ were claimed to be consistent with the picture of the Doppler-shifted local quasiparticle spectra.⁴⁰ Although somewhat less directly the experiments on thermal transport in the mixed state of cuprates^{41,42} also seem to support the basic picture of a Doppler-shifted local quasiparticle in that they are consistent with theoretical models based on this picture.^{27,31} We may thus conclude that by invoking the properties of the tunneling matrix element and lifetime effects the existing tunneling data indeed can be brought to agreement with other experiments and that the overall picture of the mixed state in cuprates appears consistent with that of a well defined qua-

siparticle propagating on a background of slowly varying supercurrents in the vortex array.

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APPENDIX: *c*-AXIS TUNNELING CONDUCTANCE IN CUPRATES

Here we illustrate how anisotropy in the tunneling matrix element $M_{\mathbf{k}}$ can emerge from the band structure predicted by standard models for the interlayer tunneling in tetragonal cuprates.^{16,17} To avoid unnecessary complications with the localized real-space wave functions in the STS tip, we outline the derivation of $M_{\mathbf{k}}$ for the case of planar interface, where we can take advantage of the fact that perpendicular momentum is conserved. Since within the model under consideration the anisotropy of $M_{\mathbf{k}}$ is essentially a consequence of the band structure of the cuprate superconductor, we fully expect the result to hold for the STS geometry. A detailed calculation indeed confirms this expectation.³⁷

According to the standard many-body formulation¹⁸ the tunneling current at a bias V between a superconductor and a normal metal is given by

$$I(V) = 2e \sum_{\mathbf{k}, \mathbf{p}} |T_{\mathbf{kp}}|^2 \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} A_S(\mathbf{k}, \epsilon) A_N(\mathbf{p}, \epsilon - eV) \times [f(\epsilon) - f(\epsilon - eV)]. \quad (\text{A1})$$

Here A_S and A_N are spectral functions of the superconductor and the normal metal, respectively, $T_{\mathbf{kp}}$ is the tunneling matrix element and e is electron charge. In the following we make the usual assumption that the normal metal in the STS tip can be described by the spectral function

$$A_N(\mathbf{p}, \epsilon) = 2\pi \delta(\epsilon - \xi_{\mathbf{p}}), \quad (\text{A2})$$

with a simple free electron dispersion $\xi_{\mathbf{p}} = (\mathbf{p}_\perp^2 + p_z^2)/2m - \epsilon_F$ where we have resolved \mathbf{p} into components parallel (p_z) and perpendicular (\mathbf{p}_\perp) to the z axis along which the tunneling current flows. The δ function in A_N can be used to perform the ϵ -integral in Eq. (A1) to obtain

$$I(V) = 2e \sum_{\mathbf{k}, \mathbf{p}} |T_{\mathbf{kp}}|^2 A_S(\mathbf{k}, \xi_{\mathbf{p}} + eV) [f(\xi_{\mathbf{p}} + eV) - f(\xi_{\mathbf{p}})]. \quad (\text{A3})$$

The expression for the tunneling matrix element can be derived from the transfer Hamiltonian formalism⁴³

$$|T_{\mathbf{kp}}|^2 = \left| \frac{\partial \epsilon_{\mathbf{k}}}{\partial k_z} \right| \left| \frac{\partial \xi_{\mathbf{p}}}{\partial p_z} \right| D(\epsilon_z) \delta(\mathbf{k}_\perp - \mathbf{p}_\perp). \quad (\text{A4})$$

Here $\epsilon_{\mathbf{k}}$ denotes the normal-state dispersion in the superconductor, $D(\epsilon_z)$ is the barrier transmission coefficient (ϵ_z is the energy of the electron inside the barrier), and the δ function reflects conservation of the perpendicular momentum under

the conditions of specular transmission. The latter can be used to carry out the summation over the transverse momentum \mathbf{k}_\perp in Eq. (A3). The remaining sum over p_z can be converted into an integral in the usual manner. The integral is then transformed by making a substitution $\omega = (p_z^2 + \mathbf{k}_\perp^2)/2m - \epsilon_F + eV$ and noting that the term $|\partial \xi_{\mathbf{p}}/\partial p_z|$ in $|T_{\mathbf{k}\mathbf{p}}|^2$ is precisely the Jacobian of this transformation; we obtain

$$I = 2e \sum_{\mathbf{k}} \left| \frac{\partial \epsilon_{\mathbf{k}}}{\partial k_z} \right| D(\epsilon_z) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A_S(\mathbf{k}, \omega) [f(\omega) - f(\omega - eV)]. \quad (\text{A5})$$

Differentiating with respect to V we finally arrive at the expression for the tunneling conductance

$$g(V) = - \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f'(\omega - eV) \sum_{\mathbf{k}} |\tilde{M}_{\mathbf{k}}|^2 A_S(\mathbf{k}, \omega), \quad (\text{A6})$$

with $|\tilde{M}_{\mathbf{k}}|^2 = e^2 |\partial \epsilon_{\mathbf{k}}/\partial k_z| D(\epsilon_z)$.

Although this last expression bears similarity to Eq. (1), the last term still contains summation over the 3D wave vector \mathbf{k} and it therefore reflects the full 3D band structure of the superconductor. In the following we shall specialize to quasi-2D cuprates and simplify Eq. (A6) further by eliminating the k_z summation, thereby expressing the tunneling conductance in terms of ab -plane properties only. To this end we consider the single particle dispersion $\epsilon_{\mathbf{k}}$ of the form deduced for tetragonal cuprates by Xiang and Wheatley:¹⁶

$$\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{k}}^\perp - t_z(\mathbf{k}_\perp) \cos k_z, \quad (\text{A7})$$

where $\epsilon_{\mathbf{k}}^\perp$ is the ab -plane dispersion depending only on \mathbf{k}_\perp and

$$t_z(\mathbf{k}_\perp) = \frac{t_z}{4} (\cos k_x - \cos k_y)^2 \quad (\text{A8})$$

is the interplane tunneling matrix element. In the following we model $\epsilon_{\mathbf{k}}^\perp$ by a free electron dispersion $\mathbf{k}_\perp^2/2m - \epsilon_F$ and noting that only values of $t_z(\mathbf{k}_\perp)$ close to the the Fermi surface are important we approximate $t_z(\mathbf{k}_\perp) \approx \tilde{t}_z \cos^2 2\phi$. While neither of these two approximations is essential for the final result, adopting them will greatly simplify the calculations. We may now convert the \mathbf{k} sum in Eq. (A6) into an integral of the form (suppressing various constant prefactors),

$$\int_{-\infty}^{\infty} dk_z \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} k_\perp dk_\perp |\tilde{M}_{\mathbf{k}}|^2 A_S(\epsilon_{\mathbf{k}}, \Delta; \omega), \quad (\text{A9})$$

where we have explicitly acknowledged the fact that $A_S(\mathbf{k}, \omega)$ will only depend on the momentum variable through $\epsilon_{\mathbf{k}}$ and $\Delta(\phi)$. We now convert the k_\perp integral into an integral over the energy, noting that, according to our assumptions, only the $\epsilon_{\mathbf{k}}^\perp$ component of $\epsilon_{\mathbf{k}}$ depends on k_\perp and thus the Jacobian of this transformation is simply a constant. We thus arrive at the final result for the tunneling conductance

$$g(V) = - \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f'(\omega - eV) \times \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\epsilon |M(\phi)|^2 A_S(\epsilon, \Delta; \omega), \quad (\text{A10})$$

where

$$|M(\phi)|^2 = \frac{2e^2 m}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \left| \frac{\partial \epsilon_{\mathbf{k}}}{\partial k_z} \right| D(\epsilon_z) = M_2^2 \cos^2 2\phi, \quad (\text{A11})$$

and M_2 is a constant. By converting the ϵ and ϕ integrals in Eq. (A10) back to a 2D \mathbf{k} -vector sum we recover the expression for the tunneling conductance written down on purely intuitive grounds in Eq. (1).

Superficially the result (A10) appears to contradict the conventional wisdom that the band structure effects are ‘‘invisible’’ to tunneling because of the exact cancellation between the $|\partial \epsilon_{\mathbf{k}}/\partial k_z|$ factors in the $T_{\mathbf{k}\mathbf{p}}$ matrix and the like factors resulting from the variable change $k_z \rightarrow \epsilon$ in the integral.^{43,44} In particular one could argue that had we done the k_z integral in Eq. (A9) first, such a cancellation would have indeed occurred, leading to the result with ϕ -independent matrix element. However, in the present case we note that the appropriate $k_z \rightarrow \epsilon$ variable change would be illegitimate because of the divergence in the corresponding Jacobian for \mathbf{k}_\perp coinciding with one of the four nodes of $t_z(\mathbf{k}_\perp)$. Therefore, as pointed out previously in connection with tunneling in cuprates,²¹ the tunneling conductance in fact *is* sensitive to nontrivial features in the band structure, such as the matrix element that vanishes along certain \mathbf{k}_\perp directions.

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